Arithmetick:

VULGAR,
DECIMAL,
INSTRUMENTAL,
ALGEBRAICAL.

In Four Parts.

Conteining

I. Vulgar Arithmetick, In *whole Numbers* and *Fractions*, in a plain and easie method.

II. Decimal Arithmetick, The ground and reason thereof, And its use illustrated by divers *Examples*.

III. Instrumental Arithmetick, Performed by *Decimal Scales*, with more ease and facility then by *Vulgar* or *Decimal Arithmetick*, all *Reductions* being wholly avoided. Nothing in this kind having been hitherto published by any

IV. Algebraical Arithmetick, Conteining an Abridgement of the Precepts of that Art, and the use thereof, illustrated by *Questions* of divers kinds.

Whereunto is added, the construction and use of several Tables of *Interest* and *Annuities*, *weights* and *Measures* both of our own and other Countries.

The third Edition, corrected and inlarged by the addition of several Rules which were not in the former Editions.

By WILLIAM LEYBOURN.

LONDON,
Printed by *J. Streater* for *George Sawbridge* living on *Clerkenwell-Green*.

TO THE READER.

HEre is presented unto thee a ſhort *Treatiſe* of *Arithmetick*, I confeſſe there are enough, (if not too many) already extant, notwithſtanding I have *adventured* to publiſh this, more for *variety* than *neceſſity*, not doubting but in the peruſal thereof thou ſhalt find ſomething in this worth thy *labour* and what in other *Books* of this kind is wanting.

The whole *Treatiſe* is divided into four Parts. The firſt conteins *Vulgar Arithmetick* in *Whole Numbers* and *Fractions* ; And in every *Rule* there are *Examples* for practice added, and *Queſtions* alſo wrought by thoſe ſingle *Rules*. In *Multiplication* I have added divers *Compendiums*, or brief wayes of *Multiplying*, whereby ſums, (having 2 or 3 figures in the Multiplier) may be performed, without any burthen or charge to the Memory, more then in ordinary *Multiplication*,

and yet no other (or at most very few) figures set down but the Product it self. And in *Division*, (which is the most difficult of the four *Species*) there are Five varieties, so that every man may make use of that which he best understands or fancies: And in the working of the *Golden Rule*, &c. I have made use sometimes of one kind of Division, and sometimes of another.

The Second Part conteins *Decimal Arithmetick*, with the ground and reason thereof, Also *Tables* of the *Moneys*, *Weights*, and *Measures* used in *England*, with directions for the making of those *Tables*, and of any other. And lastly, there are *Examples* wrought in *Decimal Numbers*, in all the most usual *Rules* of *Arithmetick*, and those *Examples* are incumbred with as many *Fractions* as can possibly happen in any *Question* concerning *buying* or *selling*.

The third Part is of *Instrumental Arithmetick*, which performeth by a new Artifice, by me contrived, any *Question Arithmetical* in a *Decimal* way, without the help of *Tables*, by which the whol work of *Reduction* is avoided, there being certain *Scales* of *English Money*, *weights*, and *Measures*, by me contrived, by which (by inspection only) the Decimal Fraction of either *Money*, *Weight*, or

or *Measure*, may be set down as exactly, and in lesse time, than they could have been taken out of the *Decimal Tables* in the Second Part of this *Treatise.* And on the contrary, any decimal fraction may be reduced into its proper parts of the integer with the same facility, speed and exactnesse. I have also in this Third Part, (the better to illustrate the use, and commodiousnesse of these *Scales*, a figure whereof is inserted at the beginning of the Third part) gone through all the most usual *Rules* of *Arithmetick*, giving *Examples* in each *Rule*, by which the *Reader* may plainly perceive what labour there is saved by using the *Scales*, the whole work of *Reduction* being taken away, and the *Fraction* immediately set down at first, without *Addition*, or being already set down, *reduced* to the known part of the *Integer*, without *Substraction.*

Unto this third part there is added an *Appendix* conteining certain Rules of *Exchanges*, with *Tables* of the *Weights* and *Measures* of *Foreign Countries* compared with the *Weights* and *Measures* used in *London*, with an Example to illustrate the use of each *Table*; And lastly, there are several Tables calculated at *6 per Cent. Compound Interest*, by which the true valuation of any

Lease or *Annuity*, or *Money forborn* or *rebated*, may be easily known, with an Example shewing the use of each Table.

The Fourth Part conteineth an *Abridgement of the Precepts of Algebra*, first written in French by *James de Billy*, a Translation whereof came to my hands some years since; In the perusal whereof, finding the *Preceps* very plain and easie, and considering that we have but very little of this kind of *Arithmetick* in our English Tongue; I have adventured to insert it here as a Fourth Part, thereby to make this *Work* the more compleat: Unto which Translation there is further added diverse *Questions* of good consequence, which were not in the *Originall*, as by comparing them together may appear.

This Treatise thus finished, I present thee with, desiring thy friendly acceptance and pardon for such faults as may possibly have escaped the Presse, or my self, and in so doing thou wilt encourage him, who is

A friend to all that are Mathematically affected,

William Leybourn.

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The Second Part of Decimal Arithmetick.

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ADVERTISEMENT

IF any Gentleman, or other Person, desire to be instructed in any of the Sciences Mathematical, as *Arithmetick*, *Geometry*, *Astronomy*, the use of the *Globes*, *Trigonometry*, *Navigation*, *Surveying of Land*, *Dialling*, or the like; Either at their own houses, his habitation, or such other convenient place as the Party shall direct, the *Author* hereof will be ready to attend them at times appointed.

Also, If any Persons would have their Land, or any Ground for Building *Surveyed*, or any Edifice or Building *Measured*, either for the *Carpenters*, *Bricklayers*, *Plaisterers*, *Glasiers*, *Joyners*, or *Masons* Work, he is ready to perform the same either for *Master Builder* or *Workman*.

Likewise, If any desire to have about his House or Garden, any kind of Sun-Dial, or Dials, of what kind soever, either fixed or movable, he will prepare or make for them such as they shall desire.

You may hear of him at the Shop of Mr. *Nathaniel Brooks* Bookseller, at the Angel in *Gresham-Colledge*, now the *Exchange*; Or at the House of Mr. *Walter Hayes*, at the Cross-Daggers in *Moorfields*, next dore to the *Popes-Head* Tavern, where you may have all sorts of Mathematical Instruments: Likewise at Mr. *Duttons* at the Sign of the Sun-Dial in *Holborn*, over against *Fetter-lane*.

VULGAR

VULGAR ARITHMETICK.

The First Part.

Numeration.

NUMERATION, is accounted the first part of *Arithmetick*, and it is to know how to read a Sum of figures expressed in writing; or to write down any Sum to be expressed.

To the doing of which there are four things necessary.

First, to know their *number*, which is *Nine*.

Secondly, their *shapes*, which are 1. 2. 3. 4. 5. 6. 7. 8. 9. Of which the first toward the left hand ever signifieth One, the second *Two*, &c.

Thirdly, to know the *value* of their *places*.

Lastly, How their proper *signification* is altered thereby.

The *Value* of their places is thus, When two, three, or more figures stand in one Summe, that is, without any *Point*, *Line* or *Comma* betwixt them,

them, as 321, that place next the right hand where the figure 1 standeth, is called the place of *Unity*, or *Unities*, and the figure 1 standeth in that place only for *one*, and the figure 2 when it is found in that first place, stands only for *two*; and the like of the rest.

But in the Summe 321, above expressed, the figure 2 is in the second place, and every place conteins the value of that place before towards the right hand ten times; and therefore the figure 2 doth not signifie *Two*, but (in this second place) ten times two, that is *Twenty*. And so the figure 3, if it had been in that place had signified ten times *Three*, that is *Thirty*; but being here in the third place, it signifies ten times *Thirty*, that is *Three hundred*. And so the whole Sum 321, is to be read, *Three hundred twenty* and *one*.

It is hereby seen, how their proper significations, which were *Three*, *Two* and *One*, are altered by being thus placed, and the Sum, which otherwise had been but *Six*, is *Three hundred twenty one*, as before.

In like sort, if there had been more places, as Seven, the value is quite through increased ten times, by being a place behind towards the left hand; as in the Sum 1111111, The figure 1 in the second place stands for ten times one (that is *Ten*,) in the third, for ten times ten (which is one *Hundred*,) in the fourth, for *Ten hundred*, (which is called One Thousand,) in the fifth, for Ten Thousand, in the sixth, for ten times *Ten thousand* (which is *One hundred thousand*) in the last, (here the seventh) place,

place, for *Ten hundred thousand*, which is called a *Million* : and so on, if there were more places, observing the same order,

Now to read this readily, Make a prick over the place of *Unity*, another over the third from it, and over every third still towards the left hand, for so those points will be over the places of *Unites*, *Thousands*, and *Millions* ; and so beginning at the last, that is, at the left hand ; read *One Million*, and because the three following towards the right, signifie properly *one hundred* and *eleven*, but the prick belonging to them is in the place of thousands, call them *one hundred* and *eleven thousand*, and the three remaining being under the point over *Unity*, signifie only *one hundred* and *eleven*; and all three points read together in one sum, is, *One Million, one hundred and eleven Thousand, one hundred and eleven.*

In like manner, if this number 73598624, were given to be read (according to former directions) make a prick over every third figure, beginning with the first figure towards the right hand, (which is the place of Unity) and then will your number stand thus.

$$\dot{7}35\dot{9}86\dot{2}4$$

Then for the ready reading thereof (because the third prick signifieth *Millions*) call all the figures towards the left hand, standing from that prick, Millions, which in this example are 7 and 3, so then this number contains 73 *Millions*, 598 *Thousand*, 624 *Six hundred twenty four*, Which in words at length we read, *Seventy three Millions, five hundred ninety eight thousand, six hundred twenty four.*

Let thus much ſuffice concerning the placing of large numbers, for the ready reading of them, only take theſe four *Tables* following, for illuſtration of what hath been hitherto delivered in words, the very ſight whereof is better then a whole Chapter of information.

The firſt Table is thus to be read] *One* in the firſt place, ſignifies *One*. *One* in the ſecond place, ſignifies Ten. *One* in the third place, ſignifies a *hundred*, &c. as in the Table.

The ſecond Table is thus to be read] In this Table you ſhall find the laſt number thereof to conſiſt of theſe figures, 357.846.903. with a point or comma betwixt every third figure, for diſtinction ſake, and alſo every three figures in their order are connected together with this brace, which denominates the places of *Millions*, *Thouſands*, *Hundreds*, ſo that the laſt number of this Table will evidently appear to be 357 *Millions*, 846 *Thouſands*, 903 *Nine hundred and three*.

The third Table] Is only certain rows of figures ſet together, and orderly diſpoſed, having the ſignification or reading of the ſame numbers in words at length to them annexed, and is only inſerted for the better ſatisfaction of ſuch as ſhall doubt whether they perfectly underſtand what hath been before taught.

The fourth Table] Is much like the ſecond, only it conſiſteth but of one number and extends three places farther then the greateſt number in the ſecond Table doth : *viz.* to twelve places ; which figures are thus to be read, 736 *Millions of Millions*, 842 *Millions*, 708 *Thouſand*, 645 *Six hundred forty five*.

(I *Table*)

(I. *Table.*)

One in the	place signifies
first	1 one
second	10 ten
third	100 a hundred
fourth	1000 a thousand
fifth	10000 ten thousand
sixth	100000 a hundred thousand
seventh	1000000 a Million
eighth	10000000 ten Millions
ninth	100000000 a hundred Million

(II. TABLE.)

								8
							5	4
						7	6	2
					3.	4	8	3
				9	7.	6	2	1
			2	4	3.	7	9·	4
		8.	7	4	9.	8	0	7
	5	7.	3	1	6.	2	4	8
3	5	7.	8	4	6.	9	0	3
Millions			Thousands			Hundreds		

(III Table.)

number of places

1	8	(eight,
2	54	(fifty four,
3	762	(ſeven hundred ſixty two,
4	3483	(three thouſand four hundred eighty three,
5	97621	(ninety ſeven thouſand, ſix hundred twenty one
6	243794	(two hundred 43 thouſand 7 hundred 94
7	8749807	(eight millions, 749 thouſ. 8 hundred and 7
8	57316248	(fifty ſeven mil. 3 hund. and.16 thouſ. 248
9	357846983	(three hundred fifty ſeven millions, eight hundred 46 thouſ. 9 hund. 83

(IIII Table.)

7	hundred	millions of millions
3	ten	
6	one	
8	hundred	millions
4	ten	
2	one	
7	hundred	thouſands
0	ten	
8	one	
6	hundred	hundreds
4	ten	
5	one	

Addition

Addition.

ADDITION, is the collecting or gathering together of two or more sums, either of one, or of divers denominations; into one sum, which is called the [*Aggregate*] [*Total*] or [*Grosse sum*.

In Addition of numbers of one denomination, the order is, to set the numbers to be added one directly under the other; that is to say, *Unites* under *Unites*; *Tens* under *Tens*; *Hundreds* under *Hundreds*; &c.

The Rule.

Having placed your numbers to be added in due order, one under another; draw a line under them, and begin at the lowermost figure towards your right hand, and add that to the next figure above, and the sum of them to the next figure above that; proceeding in this order, till you have added the whole line together; which when you have done, consider how many tens are contained in that line; and for every ten, keep one Unite *in your minde, to be added to the next row; but if there be any odd* Digits, *you must set them beneath the stroak, just under the line you added together; Having thus finished the addition of one line, proceed to the next; and from thence to the third; and so forward,*

be there never ſo many. The examples following will make this plain,

Example 1. Let the numbers given to be added together be 7832, 5609, 376, 8547, having thus placed them in order one under another as in the Margine is done; draw a line under them, then begin your addition, at the lowermoſt figure towards your right hand; Saying 7 and 6 is 13, and 9 is 22, and 2 is 24: now (becauſe in 24, there is two tens, & 4 remaining) I place the 4 under the line,

Thouſands	Hundreds	Tens	Unites
7	8	3	2
5	6	0	9
	3	7	6
8	5	4	7
22	3	6	4

and carry the two tens to the next row of *Tens*; Saying, 2 which I carried and 4 makes 6, and 7 makes 13, and 3 makes 16; in which row there is but one ten conteined, and 6 remaining, which 6 I ſet under the line, and carry the ten to the next row of *Hundreds*, ſaying, 1 that I carried and 5 makes 6 and 3 makes 9, and 6 makes 15, and 8 makes 23, in which 23, ten is contained two times, and 3 remaining; the 3 I place under the line, and carry the two tens to the next row of *Thouſands*, ſaying, 2 which I carried and 8 makes 10, and 5 makes 15, and 7 makes 22, in which, ten is conteined two times, and 2 remaining; which 2 I ſet under the line, and becauſe there is never another row to be added, to which I ſhould carry the two tens) I therefore ſet 2 down alſo under the line towards the left hand, as you ſee done in the margine: So the *totall*

totall or *grosse sum* of these numbers, being added together, is 22364.

Example 2, *A man hath in his Orchard* 136 *Apple-Trees*, 76 *Pear-Trees*, 107 *Cherry-Trees, and* 36 *Plum-Trees, and he desires readily to know how many Trees he hath in all.*

Apple-trees	137
Pear-trees	76
Cherry-trees	107
Plum-trees	35
Trees in all	355

Place your numbers one under another as in the margine, and then begin to add them together, at your right hand; Saying, 6 and 7 make 13, and 6 make 19, and 6 make 25; place 5 under the line, and carry 2 to the next row; Saying, 2 and 3 is 5, and 7 is 12, and 3 is 15, place 5 under the line, and carry 1 to the next row; Saying, 1 and 1 is 2 and 1 is 3; which 3 I set under the line, and (because there was no tens contained in that line, therefore) the total is 355, and so many *Trees* are in the *Orchard*.

Other Examples for Practice.

	95432		321		9161
	76100		1986		235
	2570		23		72
	832		1107		9
Totall	174934	Tot.	3437	Totall	9477

Additions

Addition of numbers of divers Denominations

I. Addition *of English Money.*

The most usual Coyns used in *England* are *Pounds, Shillings, Pence,* and *Farthings,* of which

4 Farthings	make	1 Peny	thus Charactred.	*d.*
12 Pence		1 Shilling		*s.*
20 Shillings		1 Pound.		*li*

For a farthing we use *q.*

THE RULE.

In the addition of numbers of divers denominations, this order is to be observed, *viz. Place all numbers of the same denomination one directly under another, as* Pounds *under* Pounds; Shillings *under* Shillings; Pence *under* Pence; *and* Farthings *under* Farthings. *Then draw a line under them, and begin your Addition with the least denomination first; Observing how many times the next greater denomination is contained in that least; and for every time carry one unite to the next denomination, as before you did the tens, setting down the remainder, if any be; Then adding the next denomination together, take notice how many times the next greater denomination is contained in that lesser; carrying for every time one to the next greater denomination.* Thus proceeding till you have gone over all the *denominations,* be they never so many. *Example*

Example 1. Let the numbers to be added together be 37 *l.* 16 *s.* 9 *d.* 3 *q.* 21 *l.* 9 *s.* 8 *d.* 1 *q.* 13 *l.* 12 *s.* 9 *d.* 2 *q.* Place the numbers as in the margine, draw a line under them, and begin with the least denomination (which in this example is *farthings*,) first, Saying,

li.	*s.*	*d.*	*q.*
37	16	9.	3.
21	09.	8.	1
13	12.	9	2
72	19	3	2

2*q.* and 1*q.* is 3 *q.* & 3*q.* is 6*q.* which is one peny and 2 *q.* remaining; whic 2 *q.* I place under the line, and carry the one peny to the next row, which is the place of pence; Saying, one peny and 9 *d.* is 10 *d.* and 8 *d.* is 18 *d.* which is 1 *s.* and 6 *d.* (*Now against the* 8, *I make a prick with my pen, for my better remembrance; to signifie, that there is one shilling to be carried to the place of shillings,*) then go on and say 6 *d.* and 9 *d.* is 15 *d.* which is 1 *s.* and 3 *d.* therefore against 9, I make a prick with my pen, and (because that is the last number) I set down the odd 3 *d.* under the place of pence, and (being I find two pricks in the line of pence, therefore) I carry 2 *s.* to the place of shillings, saying, 2 *s.* which I carried, and 12 *s.* is 14 *s.* and 9 *s.* is 23 *s.* which is one pound, and 3 *s.* remaining, I make a prick against 9, and going on, say, 3 *s.* and 16 *s.* is 19 *s.* which (being there is no more numbers to be added, and being also lesse then 20 *s.*) I set under the line, and finding one prick in the line of shillings, I therefore carry one to the place of pounds, saying, one which I carried and 3 is 4, and 1 is 5, and 7 is 12. set down the 2 under the line (as in addition of numbers of one denomination) and carry one to the next

next row; Saying, one that I carried and 1 is 2, and 2 is 4, and 3 is 7, which being the last I set down, and so the total or grosse sum is 72 *l.* 19 *s.* 3 *d.* 2 *q.*

Example 2. Let the numbers to be added be 29 *l.* 16 *s.* 8 *d.* 32 *l.* 17 *s.* 9 *d.* 81 *li.* 13 *s.* 11 *d.* and let it be required to find the totall or grosse sum.

li.	*s.*	*d.*
29	16	8.
32	17	9.
81	13	11
144	08	4

Here in this Example the least denomination is *pence*, therefore I begin with them, and say, 11 *d.* and 9 *d.* is 20 *d.* which 1 *s.* and 8 *d.* make a prick against the 9, and say 8 *d.* and 8 *d.* is 16 *d.* that is 1 *s.* and 4 *d.* make a prick against the 8, and set down the odd 4 *d.*: Then (because there are two pricks in the line of pence) you must carry 2 *s.* to the place of shillings, Saying, 2 *s.* which I carry, and 13 *s.* is 15 *s.* and 17 *s.* is 32 *s.* which is 1 *li.* 12 *s.* make a prick against 17, and say 12 *s.* and 16 *s.* is 28 *s.* make a prick against 16, and (because there is no more numbers to be added) set down the odd 8 *s.* under shillings, and (being there is two pricks in the line of shillings) carry 2 to the place of pounds; Saying, 2 and 1 is 3, and 2 is 5, and 9 is 14, set down 4 and carry 1 to the next line; and say 1 and 8 is 9, and 3 is 12, and 2 is 14, which (because it is the last) you must set down, so is the totall or grosse sum 144 *li.* 8 *s.* 4 *d.*

Other

Other Examples for Practice.

li.	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
29	18	7	3	36	2	8.
63	11	2.	1.	29	0	2
129	4	0	2	31	16.	9.
3	7	10	1	6	2	5
226	1	8	3	103	2	0

II. *Addition of* Troy-Weight.

Troy-Weight is a Weight used in *England*, by the which is weighed, *Bread*, *Gold*, *Silver*, *Pearl*, &c. the most usuall denominations of which weight are *Pounds*, *Ounces*, *Peny-Weights*, and *Grains* of which

24 Grains	make	1 Peny-weig.	thus charact.	*pw.*
20 Peny-weight		1 Ounce		*ou.*
12 Ounces		1 Pound		*lib.*

for a grain we write *gr.*

The Addition of *Troy-weight* (and consequently of any other weight or measure whatsoever, either Domestique, or Forreign) differeth nothing at all from the Addition of English Coin last taught, if the affinity of one denomination to another be first known; for whereas in money, because 12 *d.* makes

make 1 *s.* you therefore obſerve how many twelves there are in the addition of your pence, and for every 12 you add one ſhilling to the place of ſhillings, ſo in the addition of *Troy-weight*, knowing that 24 *gr.* make one *peny-weight*, you muſt therefore in the addition of *Grains* of *Troy-weight* obſerve how many times 24 you find in your line of *Grains*, and for every 24, carry one to the place of *peny-weights*, likewiſe, in the addition of *peny-weights*, you muſt conſider how many times 20 is contained in your line, and for every 20 carry one to the place of *ounces*, (becauſe 20 peny-weights make an ounce.) Alſo in the addition of *Ounces* Troy, you muſt obſerve how many times 12 you find in your line of *ounces*, and for every 12 carry one to the place of Pounds; Then laſtly, Add your Pounds together, as numbers of one denomination.

Example. Let the numbers to be added together be 7 *lb.* 11 *ou.* 13 *pw.* 19 *gr.* 6 *lb.* 7 *ou.* 16 *pw.* 19 *gr.* 3 *lb.* 7 *ou.* 9 *pw.* 6 *gr.* Place your numbers as in Addition of Money, each under other according to their reſpective denominations, as in the margine; then draw a line under them, and begin your Addition with the leaſt denomination firſt, *viz.* grains, Saying 6 *gr.*

lb.	*ou.*	*pw.*	*gr.*
7	11.	13	19
6	07.	16	19.
3	07	09	06
18	02	19	20

and 19 *gr.* is 25 *gr.* which is one peny-weight and one grain, make a prick againſt 19, and carry the odd grain to the number above, ſaying, 1 *gr.* and 19 *gr.* is 20 *g.* which (becauſe it is leſſe then one peny-weight)

peny-weight) I set under the line, then finding one prick in the line of grains, I (therefore) carry one to the place of peny-weights, saying 1 and 9 is 10, and 16 is 26, which is one ounce, and 6 *pw.* make a prick against 16, and say 6 and 13 is 19, which (being lesse then an ounce) set under the line, then for the one prick, carry 1 to the place of ounces, saying 1 and 7 is 8, and 7 is 15, which is one pound and 3 *ounces*, make a prick at 7, and say 3 and 11 is 14, which is one pound and 2 ounces, make a prick against 11, and set down the 2 ounces, and for the two pricks carry 2 pounds to the place of pounds, saying 2 and 3 is 5, and 6 is 11, and 7 is 18, which set under the place of pounds, so is your Addition ended, and the sum is 18 *lb.* 2 *ou.* 19 *pw.* 20 *gr.*

Other Examples for Practice.

lb.	*ou.*	*pw.*	*gr.*	*lb.*	*ou.*	*pw.*	*gr.*
32	9.	12	16	0	10	17.	11
17	11.	6	9.	0	6.	0	5
34	8.	15.	10	0	0	16	8.
8	10	4	7	0	5	2	19
94	3	18	18	1	10	16	19

III. *Addition of* Avoirdupois little weight.

There is another kind of weight most commonly used in *England*, called *Avoirdupois little weight*, by which is weighed all sorts of Wares or Merchandise Garblable, as *Sugar*, *Pepper*, *Cloves*, *&c.* This weight

weight is commonly divided into theſe denominations, *Pounds*, *Ounces*, and *Drams*, of which

16 Drams	make	1 Ounce	thus characted	*ou.*
16 Ounces		1 Pound		*lb.*

for a dram we write *dr.*

In the addition of *Avoirdupois weight*, you muſt obſerve the very ſame method and order as in *Money* and *Troy-weight*, having due reſpect to the quantity of the denominations, as in the addition of *drams* to make a prick at every 16, ſetting down the remainder, and for every prick carrying a unite to the next place. The preceding Rules being ſo copious in this particular I ſhall forbear to make any verball illuſtration; but only give you ſome *Examples* ready wrought, together with the moſt uſuall parts into which the *Weights* and *Meaſures* now uſed in *England* are divided: which to the ingenious will be ſufficient.

Examples

Examples *of Addition of* Avoirdupois little weights.

lb.	*ou.*	*dr.*	*lb.*	*ou.*	*dr.*
12	11.	09	06	13.	07.
76	05	12.	05	09	12
32	10.	00	06	03	09.
91	07.	13.	10	00	00
32	13.	07	05	07	09
246	00	09	34	02	05

IV. *Addition of* Avoirdupois great Weight.

There is a weight commonly used in *England*, by which is weighed all commodities that are sold by the hundred, as *Corants*, *Wool*, *Flesh*, *Butter*, *Cheese*, and the like, the which hundred weight containeth 112 pounds, and the hundred weight is divided into *Quarters*, *Pounds*, and *Ounces*, so that

16 Ounces	makes	1 Pound	thus charact.	*lb.*
28 Pounds		1 quarter of 1 C.		*qr.*
4 Quarters		1 Hundred weig.		*C.*

for an Ounce we write *ou.*

Examples *of Addition of* Avoirdupois great Weight.

C.	qr.	li.	ou.	C.	qr.	li.	ou.
37	03.	21	12	05	01	00	07
09	01	06	03	03.	02	18.	06.
33.	02	20.	00	00	01	06	08
10	00	00	00	11	03.	04	00
12	03.	07	03	06	01	10	05
103	02	27	02	17	01	11	10

I might further proceed to give you Examples of Addition of common *English Measures, viz.* of *long measures, Liquid measures*, and *Dry measures*, as also of *Time, Motion &c.* but the preceding Examples being of sufficient extent, I shall forbear to trouble either my self or the Reader with that which I conceive superfluous: Only, before I leave *Addition*, I will give you a brief view of the most usual *Measures* used in *England*, which take as followeth. And

V. *Of* Liquid Measures.

Liquid measures are those by which all sorts of Liquid substances are measured, of which (according to the Statute of 12 *Hen.* 7. *chap.* 5.) a *Pint* is the least, from which the greater *Liquid measures* are deduced, according as is expressed in the Table following.

2 Pints

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
8 Gallons		1 Firkin of Ale, Sope, or
9 Gallons		1 Firkin of Beer (Herrings
10 ½ Gallons		1 Firkin of Salmon or Eels
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel
42 Gallons		1 Tierce of Wine
63 Gallons		1 Hogshead
2 Hogsheads		1 Pipe or Butt
2 Pipes or Butts		1 Tun of Wine.

VI. *Of* Dry Measures.

Dry Measures are these by which all kind of dry substances are measured, as *Corn*, *Salt*, *Coles*, *Sand*, *&c.* of which a pint is the least.

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gollan
2 Gallons		1 Peck
4 Pecks		1 Bushel Land measure
5 Pecks		1 Bushel Water measure
8 Bushels		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey.

VII. *Of* Long Measures.

Long Measure is that by which is measured *Cloth Land*, *Board*, *Glasse Pavement*, *Tapestry*, *&c.*

of which measures (according to the Statute of 33 *Ed.* 1. and 25 *Eli.*) a *Barley Corn* is the least. So that

3 Barley Corns	make	1 Inch
12 Inches		1 Foot
3 Foot		1 Yard
3 Foot 9 Inches		1 Ell
6 Foot		1 Fadome
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ Foot		1 Pole or Perch
40 Perches		1 Furlong
8 Furlongs		1 English mile.

VIII. *Of* Time.

Time consisteth of *Years*, *Moneths*, *Weeks*, *Dayes*, *Hours* and *Minutes*. So that

60 Minutes	make	1 Hour
24 Hours		1 Day Natural
7 Dayes		1 Week
4 Weeks		1 Moneth of 28 dayes
13 Moneths, one day 6 hours.		1 Year.

IX. *Of* Apothecaries Weights.

The Weights used by *Apothecaries* are *Grains*, *Scruples*, *Drams*, and *Ounces*, of which

20 Grains

20 Grains	make	1 Scruple	thus charact.	℈
3 Scruples		1 Dram		ʒ
8 Drams		1 Ounce		℥
12 Ounces		1 Pound		℔

By help of these Tables, and the rules and cautions before expressed, any man may make addition of any of the abovesaid measures one with another, and therefore I shall forbear to illustrate them by Examples, but leave them to every mans own practice, and thus I conclude *Addition.*

The Proof of Addition.

Having placed your numbers in order, and added them together, and set the Total under the line, *Cut off the upper number by drawing a line with your pen betwixt that and the others, then add all the numbers together except the uppermost, and set the Total of them under the Total before found, then add this last Total, and the first number which you cut off with your pen together, and if the sum of those two numbers be equal with your Total sum first found, then is your work right, otherwise not.*

Example. In the first Example of whole numbers, the sums to be added were 7833, 5609, 376, and 8547, these numbers placed in due order and added together, the total or grosse sum of them is 22364, now to prove whether this

Total be true or not, I cut of the uppermost number, (to wit 7832,) with a dash of the pen, and I add the other three numbers together, namely, 5609, 376, and 8547, and the Total of them is 14532 which number being added to 7832 (the number cut off) the sum of them is 22364 exactly agreeing with the Total first found, cleerly evidencing that the Addition was truly performed; but if they had disagreed then the work had been erroneous. The like course must be taken for the proof of those sums which have different denominations as in *Money* and *Weight*, as by the examples following will appear.

	7832
	5609
	376
	8547
first Total	22364
last Total	14532
Proof	22364

Other Examples proved.

1 *Example of* Money.

	li.	*s.*	*d.*	*q.*
	37	16	9	3
	31	9	8.	1
	13	12	9	2
1 Total	72	19	3	2
2 Total	35	2	5	3
Proof	72	19	3	2

2 *Example of* Troy-Weight.

	lb.	*ou.*	*pw.*	*gr.*
	32	9	12	16
	17	11	6	9
	34	8	15	10
	8	10	4	7
	94	3	18	18
	61	6	6	2
Proof	94	3	18	18

There

There are other wayes to prove *Addition*, by casting away all the *nines* in numbers of one denomination, and of all the *twelves*, *twenties*, and *nines*, in *pounds*, *shillings* and *pence*, &c. but this as the most certain and easie I embrace, and thus much for Addition and the proof thereof.

Sub-

Substraction.

SUBSTRACTION is the taking of one or more smal sums out of a greater, as 7 *s* out of 12 *s*. or 37 *li.* out of 100 *li.* or 137 foot out of 983 foot, and the like.

As in *Addition*, the sums to be *Added* may be ~~either of one, or of divers denominations ; so~~ likewise, they may be in *Substraction*, and the manner of placing them is the same, for you must set *Unites* under *Unites*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

Example 1. Of *Substraction* of numbers of one denomination. *Let it be required to Substract* 234, *out of* 986. Place the numbers one under the other as you see done in the Margine, draw a line under them, and begin with the first figure towards your right hand, which is 4; Saying take 4 from 6, and there remains 2, place 2 under the line, and go to the next figure which is 3 ; Saying, take 3 from 8, and there remains 5, place 5 under the line, and go to the next figure which is 2 ; Saying, take 2 out of 9 and there remains 7, place 7 under the line, and your Substraction is ended, and it is evident by the work, that if you take 234, out of

Number given.	986
Number to be substracted	234
	———
Remainder.	752

of 986, there will remain 752, which you may thus prove. For if you add the 234, to 752, you shall find the sum of that addition to be 986, which is equall to the whole sum from which 234 was substracted.

Example 2. *Let it be required to substract* 2976 *out of* 96527, Place the numbers one under another as in the margine you see done, then draw a line under them, and beginning with the first figure towards your left hand; Say take 6 out 7, and there remains 1, place 1 under the line and proceed to the next figure; Saying, 7 out of 2 I cannot (wherefore you must alwayes add [10] to the number above, which in this Example is 2, and it makes it 12,) therefore take 7 out of 12, and there remains 5, place 5 under the line, and (because you added 10 to the 2 to make it 12, you must) carry a unite to the next figure; Saying, one which I carryed and 9 is 10, take ten out of 5, which I cannot, therefore I must add 10 to 5 and it makes 15, and say 10 out of 15, and there remains 5; place 5 under the line, and (because you added 10 to 5 to make it 15, you must therefore) carry a unite to the next figure: Saying, one which I carryed, and 2 is 3, take 3 out of 6 and there remains 3, place 3 under the line, and because there is no more figures to be substracted from the number above, you must, say, nothing from 9 and there remains 9, set the 9 under the line, and your Substraction is ended.

96527
2976
93551

Other

Other Examples for Practice.

	li.	*reams of Paper*			*sheep.*
Lent	5762	bought	9765	from	1000
Paid	378	sold	6529	take	394
Rests to pay	5384	unsold	3236	remains	606

Substraction of Numbers of divers Denominations,

Of English Money.

In Substraction *of numbers of divers denominations, you must observe the same order as in* Addition, *namely, to place every number in due order, with respect to the denomination, as* pounds *under* pounds; shillings *under* shillings, *&c. the greater number alwayes uppermost; And drawing a line under them, begin with the least denomination first, substracting it from the line above, and setting the* remainder *under the line as in whole numbers, but if the* pence *or* shillings *in the upper row, be smaller then those in the neather row, you must add 12 d. or 20 s. to the smaller number, that so substraction may be made, as by the* Examples *following will appear.*

Example 1. *Let it be required to substract* 38 *li.* 12 *s.* 8 *d. out of* 269 *li.* 18 *s.* 10 *d.* Place your numbers as in the Margine; then beginning with the least denomination first, (which in

	li.	*s.*	*d.*
Lent	269	18	10
Paid	38	12	8
Rest	231	6	2

in this Example is pence) Say, 8 *d.* from 10 *d.* and there remains 2 *d.* set the 2 *d.* under the the line, and proceed to the next denomination, which is shillings; Saying take 12 *s.* out of 18 *s.* and there remains 6 *s.* place 6 *s.* under the line, and go to the pounds: Saying, 8 out of 9 and there remains 1, place 1 under the line, and say 3 out of 6, and there remains 3, then (because there is no more figures to be substracted) say, nothing out of 2 and there remains 2, which set under the line, so is your substraction ended, and the remainder is 231 *li.* 6 *s.* 2 *d.*

Example 2. *Let it be required to substract* 2628 *li.* 16 *s.* 10 *d.* *out of* 9320 *li.* 10 *s.* 7 *d.*

	li.	*s.*	*p.*
Lent	9320	10	07
Paid	2628	16	10
Rests	6691	13	9

Place the numbers in order, and beginning with the pence; Say, 10 *d.* out of 7 *d.* I cannot (therefore I must add 12 *d.* (which is one shillings) to 7 *d.* and it makes 19 *d.*) but 10 *d.* out of 19 *d.* and there remains 9 *d.* set the 9 *d.* under the line, and (because I added 12 *d.* to 7 *d.*) I must therefore carry one to the place of shillings, saying 1 *s.* which I carryed, and 16 *s.* is 17 *s.* then 17 *s.* from 10 *s.* I cannot take, therefore, I must adde 20 *s.* (which is one pound) to 10 *s.* and it makes 30 *s.* then 17 *s.* out of 30 *s.* and there remains 13, set 13 under the line, and carry one to the place of Pounds; Saying, one which I carried and 8 is 9, take 9 out of 0 I cannot, but 9 out of 10, and there remains

remains 1, set 1 under the line, and carry a unite to the next place, saying 1 which I carried and 2 is 3, take 3 out of 2 I cannot, but three out of 12, and there remains 9, place 9 under the line, and carry 1 to the next place; Saying, 1 which I carry and 6 is 7, take 7 out of 3 I cannot, but 7 out of 13, and there remains 6, place 6 under the line, and carry one to the next row; Saying, 1 and 2 is 3, take 3 from 9 and there remains 6, place 6 under the line; So is your Substraction ended, and the remainder is 6691 *li.* 13 *s.* 9 *d.*

Example 3. *Suppose a man had let to another man* 1000 *pounds, and that the borrower had paid thereof at one time* 127 li. *at another time* 430 li. 10 s. *and at a third payment* 50 li. *and the Creditor would know how much he hath received, and how much is owing of his Debtor.*

Place the numbers as here you see, first the sum of mony lent, and draw a line under it; then set the sums paid at several times one under another; and draw a line under them: Then add all the sums which have been paid at several times together, which make 607 *li.* 10 *s.* which is the sum which the Debtor hath paid in all, then substract this 607 *li.* 10 *s.* from 1000 *li.* and there will

	li.	*s.*	*d.*
Money lent	1000	00	00
	127	00	00
Paid at se-	430	10	00
veral times	50	00	00
Paid in all	607	10	00
Rests to pay	392	10	00

will remain 392 *li.* 10 *s.* and ſo much is ſtill owing to the Creditor.

Other Examples for Practice.

	li.	*s.*	*d.*
Lent	2601	13	6
Paid	98	7	9
Reſts	2503	5	9

	li.	*s.*	*d.*
Owing in all	100	00	00
Paid in all	36	10	06
Reſts to pay	63	09	06

	li.	*s.*	*d.*	*q.*
Lent	3625	16	08	03
Paid at ſeveral times	100	00	00	00
	336	10	06	02
	039	12	09	02
	100	00	00	00
Paid in all	576	03	04	00
Reſts to pay	3049	03	04	03

The Proof *of* Subſtraction.

The *Proof* of *Subſtraction* is performed by *Addition*, for adding the number to be ſubſtracted to the remainder, the Sum of them muſt be equal to the number given, if you have truly wrought. As in the firſt example of numbers of one denomination,

The number given is	986
The number to be ſubſtracted is	234
The remainder is	752
Proof	986

Add

Add the number to be substracted 234, to the remainder 752, the sum of them is 986, equal to the number given.

Examples for Practice proved.

	li.	*s.*	*d.*		*li.*	*s.*	*d.*
Lent	62	18	09	Borrowed	100	00	00
Paid	37	19	06	Received	36	13	04
Rests	24	19	03	Due	63	06	08
Proof	62	18	09	Proof	100	00	00

Other Examples in Weight *and* Measure.

1 Exam. in *Troy weight*

Bought	*li.*	*ou.*	*pw.*	*gr.*
of Silver	07	11	13	19
Sold	05	07	03	05
unsold	02	04	10	14
Proof	07	11	13	19

2 Example in *Avoridupois great weight.*

	C.	*q.*	*li.*	*ou.*
Bought	37	03	22	11
Sold	13	01	23	06
Rests	24	02	27	05
Proof	37	03	22	11

3 Example in *Avoirdupois little weight.*

	li.	*ou.*	*dr.*
Bought	84	12	13
Sold	26	08	11
Rests	58	04	02
Proof	45	12	13

4 Example in Time.

	dayes	*ho.*	*m.*
From	364	23	50
Take	76	09	22
Rests	288	14	28
Proof	364	23	50

Que-

Questions performed by Addition *and* Substraction.

Question 1. *What number is that which being added to 376 shall make 1000?* Substract 376 from 1000, the remainder is 624, the number sought.

Question 2. *What number of Pounds, Shillings and Pence must be added to 36 li. 17 s. 3 d. to make that sum up 100 l.* Substract 36 *li.* 17 *s.* 3 *d.* from 100 *li.* the remainder is 63 *li.* 2. *s.* 9. *d.* which added to 36 *li.* 17 *s.* 3 *d.* makes 100 *li.*

Quest. 3. I*n the year of our Lord 1440, the famous art or mystery of Printing was invented, I would know how long it is since that time to this year of our Lord 1655.* From 1655, substract 1440, the remainder is 215, and so many years are expired since *Printing* was invented.

Question 4. *An Army consisting of 13721 Horse and 26850 Foot, in an engagement there were slain 3760 Horse and 7523 foot; the question is how many were slain in all, and how many Horse and how many foot escaped.* From the 13721 Horse which went out, substact the 3760 that were slain, there remains 9961, and so many Horse escaped; Also from the 26850 Foot which went out, substract the 7523 which were slain, and there remains 19327, the number of Foot which

35 · 17 3
63 2 9
100 – 0

which escaped, & by adding the 3760 Horse which were slain, to the 7523 Foot that were slain, their Total is 11283, and so many were slain in all.

Multi-

Multiplication.

MUltiplication is that part of *Arithmetick* which teacheth how to encreaſe one number by another, ſo that the number produced by their *Multiplication* ſhall contain one of the numbers multiplied, ſo many times as there are *unites* contained in the other. *Multiplication* may fitly be termed a Compendium of *Addition*, for that it performeth at one operation the ſame, which to effect by Addition, would require many. For inſtance, if it were required to know how much 7 times 5 is, to perform this by Addition, I muſt ſet ſeven fives, or five ſevens, one under another, and adding them together, I ſhall find that either of their Totalls ſhall contain 35, but this by *Multiplication* is performed with far more brevity, as by *Examples* hereafter ſhall appear.

Before you enter upon the practice of *Multiplication*, it is neceſſary to remember the product produced by the multiplication of any one of the nine Digits, by any other of the ſame, as readily to know; that 4 times 5 is 20, 6 times 7 is 42, 2 times 9 is 18, 7 times 9 is 63, 8 times 9 is 72, &c. Which this Table following will plainly declare, and

must be perfectly learned by heart, before you attempt to multiply great numbers.

Multiplication Table.

$$2\text{ times}\left\{\begin{matrix}2\\3\\4\\5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}4\\6\\8\\10\\12\\14\\16\\18\end{matrix}\right\}\quad 3\text{ times}\left\{\begin{matrix}3\\4\\5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}9\\12\\15\\18\\21\\24\\27\end{matrix}\right\}\quad 4\text{ times}\left\{\begin{matrix}4\\5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}16\\20\\24\\28\\32\\36\end{matrix}\right\}$$

$$5\text{ times}\left\{\begin{matrix}5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}25\\30\\35\\40\\45\end{matrix}\right\}\quad 6\text{ times}\left\{\begin{matrix}6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}36\\42\\48\\54\end{matrix}\right\}\quad 7\text{ times}\left\{\begin{matrix}7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}49\\56\\63\end{matrix}\right\}$$

$$8\text{ times}\left\{\begin{matrix}8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}64\\72\end{matrix}\right\}\quad 9\text{ times } 9 \text{ makes } 81.$$

The use of the Table of Multiplication, *and the manner how it is to be read.*

This Table sheweth what the Sum of any two digits multiplyed one by another doth amount unto, and is thus to be read, 2 times 2 makes 4, 2 times 3 makes 6, 2 times 4 makes 8: Also 6 times 4 makes 24, 7 times 8 makes 56, 8 times 8 makes (or is) 64, 9 times 9 is 81, &c. Ano-

Another Table of Multiplication.

1	9	8	7	6	5	4	3	2	1
2	18	16	14	12	10	8	6	4	
3	27	24	21	18	15	12	9		
4	36	32	28	24	20	16			
5	45	40	35	30	25				
6	54	48	42	36					
7	63	56	49						
8	72	64							
9	81								

THis Table is thus to be read, In the first Row or Column towards the left hand, and also at the top of the Table you have the nine *Digits* in bigger Figures then the rest; the Figures in the first Column beginning with 1, and so proceeding by 2. 3. 4, &c. to 9. Those at the top of the Table beginning with 9 towards the left hand, and so backwards by 8, 7, 6, &c. to 1 at the right hand.

Now if by this Table, you would know how much 8 times 7 is, find 8 among the great Figures at the head of the Table, and look down that *Row* or *Column* till you come against 7 of the great Figures in the first *Column*, against which you shall find 56, and so much is 8 times 7, or eight multiplyed by 7.

In the ſame manner may you find that 7 times 9 is 63, 5 times 6 is 30, 3 times 4 is 12, and ſo of any two of the nine Digits.

In Multiplication there are three terms commonly uſed, that is to ſay;

The *Multiplicand*,
The *Multiplier*, and
The *Product*.

The *Multiplicand*, is the number to be multiplied.

The *Multiplier*, is the number by which the Multiplicand is multiplied: and

The *Product*, is the number which is produced by the multiplication of the Multiplicand and the Multiplier together.

Thus, if it were required to multiply 8 by 7, here 8 is the *Multiplicand*, 7 the *Multiplier*, and 56 is the *Product*, for 8 times 7, or 7 times 8 is 56.

In *Multiplication* it mattereth not which of the two numbers is made the *Multiplicand*, or which the *Multiplier*, for the *Product* produced by either will be the ſame; but the uſual way is to make the *greater* number the *Multiplicand*, and the *leſſer* number the *Multiplier*.

THE RULE.

The numbers to be multiplied muſt be ſet one under another, viz. *the* Multiplicand (*or greater number*) *above, and the* Multiplier (*or leſſer number*) *below, the laſt figure of the* Multiplier *under the laſt figure*

figure of the Multiplicand; *then draw a line under them, and (having learned the preceding Tables perfectly by heart) multiply every digit of the* Multiplier, *into every digit of the* Multiplicand, *setting the several Products under the line; Then having finished your* Multiplication, *draw a line, and add all the products together, and the Sum of those products is the general* Product *of the whole multiplication, as by the following* Examples *will appear.*

Example 1. *Let it be required to multiply* 736 *by* 7. First, I write down 736 the Multiplicand, and under it 7, the Multiplyer, and under them I draw a line, then I multiply 7 into every digit of the Multiplicand; Saying, 7 times 6 is 42, place 2 under the line, under 7, and for the four tens keep 4 in mind; then say again, 7 times 3 is 21, and 4 which I kept in mind is 25; place 5 under the line, and keep the two tens in mind; then say again, 7 times 7 is 49, and 2 which I kept in mind is 51; place 1 under the line, and the 5 tens kept in mind (because there is no more figures to be multiplied) I set down under the line also, so is the work ended, and the product of this multiplication is 5152.

736	*Multiplicand*
7	*Multiplier*
5152	*Product.*

Example 2. *Let it be required to multiply* 3417 *by* 5. Place the numbers one under another, and draw a line under them as in the margine; then begin you multiplication, Saying, 5 times 7 is 35, place 5 under the line, and keep the three tens in mind,

mind, then say again, 5 times 1 is 5, and 3 which I kept in mind is 8, place 8 under the line, and (because it is lesse then 10, I keep nothing in mind) then say again, 5 times 4 is 20, place a cypher under the line, and keep the two tens in mind; lastly, say 5 times 3 is 15, and 2 which I kept in mind is 17, which 17 (being the last number) I place under the line, and so is my Multiplication ended, and the *Product* is 17085.

```
 3417
    5
-----
17085
```

¶ You may be satisfied of the truth of this work, if you will take the pains to set down the Multiplicand 3417, five times one under another, and add them together, as so many several sums, so shall you find the *Total* of that addition, to be 17085, exactly the same with the Product of this multiplication.

Example 3. In the two fore-going examples, the *Multiplier* consisted but of one *Digit*, we are now to shew how *Multiplication* is performed when the multiplier consists of more then one figure, therefore in this Example, *Let it be required to multiply* 5704 *by* 37. Place your numbers, and draw a line under them as you see in the margine; Then begin your multiplication in this manner; Saying, 7 times 4 is 28, set 8 under the line, and

```
  5704 Multiplicand
    37 Multiplier
------
 39928
17112
------
211048 Product
```

keep

keep the two tens in mind, then ſay 7 times nothing is nothing, but the two tens in mind is 2, ſet 2 under the line, then ſay 7 times 7 is 49, ſet 9 under the line, and keep 4 in mind, then laſtly, ſay 7 times 5 is 35, and 4 in mind is 39, which being the laſt number to be multiplyed I ſet down under the line, ſo is the multiplication of one of the digits (namely 7) finiſhed.

Then begin to multiply the ſecond digit, ſaying 3 times 4 is 12, place 2 in the ſecond line, one place towards the left hand, and keep 1 in mind, then ſay 3 times nothing is nothing, but 1 in mind is 1, ſet down 1 by the 2 in the ſecond line; thirdly, ſay 3 times 7 is 21, place 1 in the ſecond line, and keep the two tens in mind: Laſtly, ſay 3 times 5 is 15, and 2 in mind is 17, which 17 (becauſe there is no more figures to be multiplied) I place in the ſecond line alſo.

Having thus done, I draw a line under them, and add theſe two lines together, as in common Addition of numbers of one denomination; Saying 8 is 8, place 8 under the line; then ſay 2 and 2 is 4, place 4 under the line; then ſay 1 and 9 is 10, place a cypher under the line, and carry 1 to the next place: ſaying, 1 and 1 is 2 and 9 is 11, place 1 under the line, and carry 1 to the next row, ſaying 1 and 7 is 8, and 3 is 11, place 1 under the line, and carry 1 to the next place; ſaying, 1 which I carry and 1 is 2, place 2 under the line, and ſo is your multiplication ended, and the product is 211048.

Example 4. *Let it be required to multiply* 57325 *by* 4032. Place the multiplicand and the multiplier one under another, and drawn a line as before; then proceed to the multiplication as formerly: saying, First, 2 times 5 is 10, set down a cypher, and keep 1 in mind: then 2 times 2 is 4, and 1 in mind is 5, place 5 under the line: then 2 times 3 is 6, set 6 under the line: then 2 times 7 is 14, set down 4, and keep 1 in mind; then 2 times 5 is 10, and 1 in mind is 11, which 11 (being the last) I set down.

57325	Multiplicand
4032	Multiplier
114650	
171975	
229300	
231134400	Product

The multiplication of one of the digits being finished, proceed to the multiplication of the next; Saying, 3 times 5 is 15, set down 5 in the second line, a place more towards the left hand, and keep 1; then 3 times 2 is 6, and 1 kept is 7, set down 7; then 3 times 3 is 9, set down 9; then 3 times 7 is 21, set down 1 and keep 2 in mind; then 3 times 5 is 15, and 2 in mind is 17; which being the last set down also.

Two of the figures of the multiplier being finished, proceed to the third, which (in this example) being a cypher, you may wholly neglect, and proceed to the multiplication of the fourth figure, onely remember to remove the Product of the fourth figure one place more to the left hand, as in the example you may see, for the cypher, though

though it be not written down, yet it muſt keep its place, and the figures following muſt be removed a place farther.

Then for the Multiplication of the fourth and laſt digit, ſay 4 times 5 is 20, ſet down a cypher (under 9) and keep 2 in mind: then 4 times 2 is 8 and 2 in mind is 10, ſet down a cypher, and keep 1 in mind: then 4 times 3 is 12, and 1 is 13, ſet down 3, and keep 1: then 4 times 7 is 28, and 1 kept is 29, ſet down 9, and keep 2: then 4 times 5 is 20, and 2 kept is 22; which 22 (becauſe the multiplication is ended) ſet down alſo.

Having thus multiplied all the digits ſeverally, draw a line under their Products, and adde them altogether as in the former Example, ſo ſhall you find their general Product to be 231134400.

Other Examples for Practice.

```
     73260           50762
     45003            4567
  ----------      ----------
    219780          355334
  366300           304572
 293040           253810
 ----------      203048
 3296919780      ----------
                 231830054
```

The

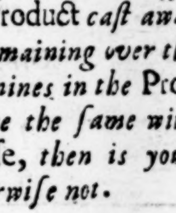
The Proof of Multiplication.

The most certain proof of *Multiplication* is by *Division*, but because *Division* is not yet known, I will here shew a near way by which *Multiplication* may be proved. Which is thus,

THE RULE.

Make a Crosse, *as in the Margine, then, any sums being multiplied, you may prove the truth of your work in this manner,* (1) *Cast away all the nines which you can find in the* Multiplicand, *what remaineth set on the right side of the* Crosse. (2) *Cast away also the nines in the* Multiplier, *and what remains set on the left side of the* Crosse. (3) *Multiply the figure on the right side of the* Crosse, *and out of that* Product *cast away the nines also, setting the figure remaining over the* Crosse, *then* (4) *Cast away all the nines in the* Product, *and if the figure remaining be the same with that which standeth over the* Crosse, *then is your multiplication truly performed, otherwise not.*

```
 4324
   23
-----
12972
8648
-----
99452
```



Example 1. Let it be required to prove the Sum in the margine.

1. Cast away all the nines in the *Multiplicand*; Saying, 4 and 3 is 7, and 2 is 9, which being rejected, there remains 4, which I set on the right side of the crosse, Then

2. Cast

2. Cast away all the nines in the *Multiplier*: Saying, 2 and 3 is 5, (which being lesse then 9) I set on the left side the Cross. Then

3. Multiply 4 by 5, saying 4 times 5 is 20 from which cast all the nines which are two, and there remains 2, place 2 over the Cross. And

4. Cast away all the nines in the *Product*; Saying, 2 and 5 is 7, and 4 is 11, cast away 9, and there remains 2 : which exactly agrees with the figure over the *crosse*, and demonstrates that the Multiplication is truly performed.

Compendiums in Multiplication.

1. If the *Multiplier* consist of cyphers in the last place or places, you may omit the multiplication of them, and place the former figures of the Multiplier under the Multiplicand: Thus, if it were required to multiply 3257 by 2600, place the numbers as you see in the margent, then multiplying 3257 by 26, the Product will be 84682, to which if you add two cyphers, (because there were two cyphers in the Multiplier) it will be 8468200, which is the true product of the multiplication.

```
   3257
   2600
 -------
  19542
  6514
 -------
8468200
```

2. If it be required to multiply any number by 10, 100, 1000, 10000, *&c.* You have no more to do, but to add so many cyphers to the *Multiplicand*, as there are cyphers in the *Multiplier*, Thus if you were to multiply 365 by 10, the product will be 3650, or by 100, it would be 36500, or by 1000, it would be 365000, or by 10000, it would be 3650000.

3. If any number given were to be multiplied by 5, you may abbreviate your work thus. Add a cypher to the *Multiplicand*, take half that number, and it shall be the product required. Thus if it were required to multiply 8727 by 5, add a cypher to the Multiplicand, then it is 86270 the half whereof is 43135, which is the product required.

To multiply by any of the nine Digits without charging the memory.

To multiply any number by 2, Either double the number in your mind, or add it, by setting it down twice, so 57325 produceth 114650.

To multiply any number by 3, To the number given, add the double thereof, the sum is the product, so 57325 produceth 171975.

To multiply any number by 4, Double the duplication in your mind, so 57325 produceth 229300.

To multiply any number by 5, Conceive a cypher added to the given number, and in your mind half thereof is the product, thus a cypher added to 57325, maketh it 573250, the half whereof is 286625.

To multiply any number by 6, Take half adding a cypher, and add to the half, the figure standing next before.

before. Thus 573250 produceth 343950.

To multiply any number by 7. Take half and add it to the double of the former figure, ſuppoſing a cypher added as before; So 57325 thus ordered, produceth 401275.

To multiply any number by 8. Double each former figure, and ſubſtract it from the following, ſo 57325 produceth 458600.

To multiply any number by 9. Suppoſe the number multiplyed by 10, then ſubſtract each former figure from the following, beginning with that next before the cypher, the remainder is the product, ſo 57325 produceth 515925.

Other Brief Rules of Multiplication.

Having ſhewed you ſome *Compendious* wayes of *Multiplying* by *Article numbers*, as by 10, 100, 1000, *&c.* and alſo by the *nine Digits*, without any trouble or charge to the *memory*: I will now ſhew how you may expeditiouſly and certainly, *multiply* any ſum by divers other Numbers conſiſting of *two* or *three* places, without ſetting down many figures but the Product it ſelf.

To multiply any Number by 11.

THE RULE.

Set the Multiplicand down twice, removing it one place to the left hand, add them together, the ſum is the Product of the Number multiplyed by 11.

Example. *Let it be required to multiply* 97 *by* 11.

Set

Set down 97 twice, removing one of them a place more to the right hand as you see here in the margine, then add them together, the sum is 1067, which is equal to the Product of 97 multiplyed by 11.

97
97
1067

To multiply any Number by 12, 13, 14, 15, 16, 17, 18, *or* 19.

The Rule.

To effect this, You have no more to do, but to *Multiply the given Number by* 1, 2, 3, 4, 5, 6, 7, 8, *or* 9, *and in your Multiplication, continually to add that figure of the Multiplicand which standeth on the right hand of the figure you are multiplying by, setting down the sum for the figure of the Product.*

Example. *Let it be required to multiply* 3624 *by* 17.

Multiply in this manner by 7, Saying, 7 times 4 is 28, set down 8 and carry 2 —— then 7 times 2 is 14, and 2 which I carryed is 16, and 4 *(the figure of the Multiplicand which stands on the right hand of* 2) is 20 : Set down 0, and carry 2 —— then 7 times 6 is 42, and 2 which I carryed is 44, and 2 *(which stands on the right hand of* 6) is 46; set down 6 and carry 4 —— then 7 times 3 is 21, and 4 which I carryed is 25, and 6 (*which stands on the right hand of* 3) is 31, set down 1, and carry 3, which 3 add to 3 (*the left hand figure of your Multiplicand,*) it is 6 which set down, so is the Product of

of 3624 multiplyed by 17 is 61608.

In the ſame manner, may you multiply by 12, 13, 14, *&c.* as in theſe Examples

3764	625	4793
13	16	19
48932	10000	91067

To multiply any Number by 102, 103, 104, 105, 106, 107, 108, *or* 109.

THE RULE.

Multiply the Number given by 2, 3, 4, 5, 6, 7, 8, *or* 9, *ſetting the Product two places towards the right hand of the Multiplicand, the Product and the Multiplicand added together in the ſame order that they ſtand, ſhall be the Product of the whole Multiplication.*

Example. *Let it be required to multiply* 3624 *by* 106.

Set them down as in the Margine, then multiply 3624 by 6, it produceth 21744, which added to 3624, in the ſame order as they there ſtand, the ſum of that Addition will be 384144, which is equall to the Product of 3624 multipled by 106

```
 3624
  106
------
  21744
------
 384144
```

Other

Other Examples.

765	6374
103	107
2259	44618
78759	682018

To multiply any number by 112, 113, 114, 115, 116, 117, 118, *or* 119.

THE RULE

Multiply the given Sum by 12, 13, 14, 15, 16, 17, 18, *or* 19, *as hath been showen already, setting the Product two places to the right hand of the Multiplicand, then add this Product and the Multiplicand together in the same order as they stand, so shall the sum of that Addition be equall to the Product of the Multiplication.* As by the Examples following is evident.

Multiply	4065
By	113
The Product mul. by 13	52845
Real Product	459345

Multiply	7632
By	119
The Product mul. by 19	145008
Real Product	908208

Questions

Questions performed by Multiplication only.

Question 1. *If a piece of Land be 236 Perches long, and 182 Perches broad, how many square Perches are contained therein?* Multiply 236 the length, by 182 the bredth, the product is 42952, and so many square Perches are contained in such a square piece of Land.

Question 2. *In a yeer there are 365 dayes natural, and in every day 24 hours, how many hours be there in a yeer?* Multiply 365 the number of daies by 24 the number of hours, the product is 8760, and so many hours be there in a yeer.

Question 3. *From* London *to* Coventry *it is accounted 76 miles, how many yards therefore is it from* London *to* Coventry? Multiply 1760 (which are the number of yards contained in one mile) by 76, the product is 133760, and so many yards are between *London* and *Coventry*.

Division.

DIVISION is the just contrary to *Multiplication*, for *That* turns *Small* denominations to *Greater*, as *Multiplication* turns *Greater* to *Smaller*. Or (in whole Numbers, of which only we yet speak) *Divi-*

ſion, is the asking, *how many times one Sum is contained in another*? and the number which anſwereth to that queſtion is called the *Quotient*.

And the Number containing, which is to be divided, is called the *Dividend*.

And the Number contained, or by which the Dividend is to be divided, is called the *Diviſor*.

And as often as the *Dividend* contains the *Diviſor*, ſo often doth the *Quotient* contain *Unity*.

The wayes of performing Diviſion are divers, I will begin with that which is moſt uſed and taught, which is as followeth.

THE RULE.

Place the Diviſor *under the* Dividend, *ſo that the figures next to the left hand ſtand directly one under the other, if the reſt of the Diviſor be not the greater: or if all the* Diviſor *be greater then that above it, then the ſaid* Diviſor *muſt be devolved one place further toward the right hand; having ſo placed them, try how many times the lower figures are contained in the upper figures, and write that figure which anſwereth that queſtion within a crooked line in the margine of the work, which is called the* Quotient, *and by it multiply the firſt figure of the* Diviſor, *and take the Product out of the figures directly over it, beginning the* Subſtraction *towards the left hand; then cancel that figure of the* Diviſor, *and alſo, that of the* Dividend *which hath been already uſed, with a light daſh of a Pen, and write the* remain (*when the Product of the firſt figure multiplied by the* Quotient *is ſubſtracted as before*) *juſt over the figure uſed and cancelled*; *Then*

Then proceed to do the like with the second, third, and fourth figure of the Divisor *if there be so many; till having cancelled it all, and set the remain orderly above the* Dividend, *you have finished one work.*

Now if the Dividend *have still some figures untouched towards the right hand, then remove the* Divisor *still towards the right hand, but one place at a time, and then again ask or try how many times the lower may be had in the upper, and write the answer in the* Quotient, *whether it be* 1 *or more, (only it cannot be above* 9*) or nothing, then put* 0 *in the* Quotient, *and multiply the* Divisor *by this new figure, and substract the Product, setting the remainder orderly above, as before, this work must be repeated by removing the* Divisor *still one place towards the right hand, until* Unity *in the* Divisor *stand under* Unity *in the* Dividend, *and then the work is done.*

Example 1.

Let it be required to divide 4096 by 3
Place them thus 4096 (1
3

Ask how many times 3 in 4, the answer is 1, which is put within a crooked line by it self.

Then in your mind multiply the Divisor 3 by the quotient 1. And having said these words, *Once* 3 *is* 3, presently cancel the 3. And having added these words, *Out of* 4, cancel the 4. And after these words, *and there remains* 1, write 1 just over the 4, as you see here done.

1
~~4~~096 (1
~~3~~

 Then

Then remove the Divisor one place towards the right hand, saying, *how many times 3 in* 10, the answer is 3, which write in the quotient; and in

~~1~~1
~~4~~~~0~~96 (13
~~3~~~~3~~

your mind multiply the Divisor 3 by the quotient 3, the product is 9, wherefore say, three times three is nine, out of ten, and there remains one, then (having cancelled the 10 & the 3,) write over them 1.

Again, remove the Divisor 3 one place more, asking *how many times 3 in* 19? the answer being 6, write 6 in the quotient, and say 6 times 3 is 18 out of

~~1~~~~1~~1
~~4~~~~0~~~~9~~6 (136
~~3~~~~3~~~~3~~

19, and there remains 1, wherefore having cancelled the 3, and the 19, write 1 over 3, and remove the Divisor once more, and ask *how many times 3 in* 16? answer is 5, which write in the quotient, then in mind say 5 times 3 15 out of 16, and there remains 1, and cancelling the 16 and the 3, write over 3, 1: Now because the *Divisor* 3, is advanced so far

~~1~~~~1~~~~1~~1
~~4~~~~0~~~~9~~~~6~~ (1365
~~3~~~~3~~~~3~~

till it it is come to stand under 6 in the *Dividend*, which 6 is the place of *Unity* there, the said *Divisor* cannot be removed any more, and therefore the Division is ended, and the *Quotient* being 1365, shews that the *Divisor* 3 is contained in the *Dividend* 4096, 1365 times; and 1 remaining, which 1 being lesse then the Divisor 3, doth not contain it once, but one third part of once, which (after the Reader hath skill in broken numbers) must be joined to the quotient thus 1365 $\frac{1}{3}$.

The

The best proof of this *Division* is by *multiplying* the quotient into the Divisor, and to the Product add the remain; then if the work be well done, the sum shall be equal to the Dividend.

So 1365 multiplied by 3 produceth 4095, to which adding the remain 1, the sum 4096 is equal to the Dividend.

Here we divided only by one figure 3, because the first Example being easie and clear, should be a fair Introduction to the second.

Note, that if the Divisor had been greater then 4, as 5, the work must have begun thus 4096 (
5

So the quotient would have been 819 $\frac{1}{5}$, which is one place lesse.

Example 2.

Let it be required to divide 1310720 by 4096,

Place them thus 1310720 (
4096

Now the question to be asked is, how many times 4096 is there in 13107? to find an answer to this question, the Reader which hath but an indifferent faculty of judging, may do it, (for the most part) by considering the first figure in the Divisor, as here 4, for presently he knows that 4 times 4 is 16, which cannot be had in 13, therefore the first figure in the quotient must be lesse then 4.

Again, it cannot be much lesse, because the second place in the Divisor is 0.

He may therefore venture on 3, And having put 3 in the quotient, say 3 times 4 is 12, which substract out of 13 over it thus, say 2 out of 3, and there remains 1, and 1 out of 1 and there remains 0, cancelling the 0 and the 13, and set the remain 1 over the 4, as you see.

$$
\begin{array}{cccccccc}
 & & & 1 & & & & \\
 & \cancel{1} & 8 & \cancel{3} & 9 & & & \\
\cancel{1} & \cancel{3} & \cancel{1} & \cancel{0} & \cancel{7} & 2 & 0 & (3 \\
 & \cancel{4} & \cancel{0} & \cancel{9} & \cancel{6} & & &
\end{array}
$$

Then go on saying 3 times 0 is 0, out of 11, and remains still 11, again 3 times 9 is 27, take 7 out of 10 there remains 3, to be set over the 0, which is over 9, and 2 with 1 borrowed (to make the 0 10, from the which the 7 was taken) is 3, and 3 out of 11, remains 8, which write over the place of 0 in the Divisor, cancelling the 9 in it, and also those figures of the Dividend 110, out of which you have taken any thing; then lastly, say 3 times 6 is 18, take 8 out of 7 I cannot, therefore (borrowing 10,) say 8 out of 17 remains 9, which write over the 7, then in the next place, the 10 borrowed is 1, and the 10 in the 18 is 1. Say therefore 1 and 1 is 2, out of 3, and the remain is 1, which write over the 3, having still cancelled the figures which you have used orderly as you go.

Now the whole Divisor being cancelled, it must be removed on one place further, and placed as here.

Now

Now ask again, how many times 4096 in 8192? it will be found (because of the 0 after the 4) as often as 4 in 8, that is two times, put therefore in the quotient 2, and work as before, saying 2 times 4 is 8, out of 8 remains 0, then 2 times 0 is 0 out of 1 remains 1, then 2 times 9 is 18, take 8 out of 9 remains 1, (which put over the 9,) and 1 out 1 in the next place remains 0, then lastly, 2 times 6 is 12, that is, 2 out of 2 there remains 0, and 1 out of 1, there also remains 0, cancel and put the remainder over as formerly.

00
011
018390
1310720 (320
409666
4099
40

Now again, the Divisor being all cancelled, should be removed; and ask how many times 4096 in 0000, the answer is nothing 0, which being being put in the quotient, the work is all done.

And the quotient 320, shews that the *Divisor* 4096 is contained in the *Dividend* 1310720, three hundred and twenty times.

And whether the Divisor have 2, 3, 5, 7, or more places, the working is still like this, not differing from it at all.

Compendiums in Division. For brevity in some cases, where the *Divisor* towards the right hand hath one *Cypher* or more; those Cyphers may be placed orderly under the *Dividend* at first, and remain there till the work be done, with the rest of the *Divisor*, which must needs shorten the Division.

As if 2587645 *were to be divided by* 15000,

 place

place them thus:

```
2587645
15  000
```

And ſay how many times 15 in 25? anſwer is 1, which write in the quotient, and multiply in mind the Diviſor by the quotient, ſaying once one is one, out of 2 there remains 1, then cancel the 2, and the 1 under it, and write the remaining 1 over it, as here is done, then ſay once 5 is 5 out of 5, and there remains 0, therefore cancel the two 5, and over the uppermoſt write 0.

```
   0 0
    3 1
  1 0 3 (7
  2 5 8 7 (6 4 5 (172
  1 5 5 5 0 0 0
    1 1
```

Now remove the Diviſor one place, and ask how many times 15, in 108? anſwer is 7, which write in the quotient, and multiply, ſaying 7 times 1 is 7, out of 10 remains 3, which write over the 0, then ſay 7 times 5 is 35, take 5 out of 8 remains 3, which write over 8, and 3 out of 3 remains 0.

Now again, remove the Diviſor, and ask how many times 15 in 37? anſwer is 2, which write in the quotient, and ſay two times 15 is 30, out of 37 remains 7, and the Diviſion is ended, the remain being 7645.

Proof

Proof of this.

Multiply the quotient	172
by the Divisor	15
	860
	172
The Product is	2580

Before which put the three Cyphers, And then it is 2580000
To which add the remain 7645

The Total is 2587645

Which is equal to the Dividend, and therefore the work is right.

So if one would divide any sum by 10, 100, 1000, 10000, &c. he need but cut off the first, two first, three first, or four first figures towards the right hand, the other figures shall be the quotient, and those cut off, the remain.

As if 2587645 be divided by 1000, the quotient is 2587, and the remain 645.

If you would know how many pounds Sterling are in 95670 shillings; having placed them thus, that is, 2 under 9, and 0 under 0, then divide by 2, which is very easie. So the quotient 4783, shews that in the number of shillings given, is contained 4783 pounds, and 10 shillings remaining.

$\cancel{1}\cancel{1}$ (1
$\cancel{9}\cancel{5}\cancel{6}\cancel{7}$ (0 (4783
$\cancel{2}\cancel{2}\cancel{2}\cancel{2}$ 0

The

The reason why the Divisor was 2, is because there are 20 shillings in one pound, and therefore any number of shillings is turned into pounds, by dividing by 20, that is by 2, putting the cypher under unity, onely to fill a place at last. And this way of turning any number of shillings into pounds, may be easily effected by memory, if you suppose the last figure of your given number to be cut off with a line or comma, and taking the half of the other figures. Thus, let the given number of shillings be 5739; Imagine the last figure 9 to be separated from the rest by a line thus 573 | 9; now by memory take the half of 573, by saying in your mind the half of 5 is 2, (and one remaining which makes the 7 following 17) the half of 17 is 8 (and one remaining) the half of 13 is 6 (and 1 remaining, which 1 is 10 *s.* to be added to the 9, and the whole is 286 *li.* 19 *s.*

In this way of Division (as in all others) if the *Remain* at last be greater then the *Divisor*, the *Quotient* is not just, but too little, which may be remedied (without beginning the work again) by dividing the remainder onely by the same Divisor, for thereof will arise a new quotient, which added to the former quotient, the sum will be the just quotient.

So if 7290 be divided by 27, the numbers being placed thus,

Because 27 can be had in 72 but twice, put 2 in the quotient, saying 2 times 27 is 54, out of 72, and there remains 18, which write over 72, cancelling as before is shewed; then removing the Divisor, say how many times

2
59
1847
7290 (259
2777
22

times 27 in 189? anſwer is 7, but if one ſhould miſtake, and write 5 in the quotient, and ſay 5 times 27 is 137, out of 189, remains 54, and write it over as before; and remove the Diviſor, and ſay, how many times 27 in 540? anſwer is 20, but ſhould not be above 9, ſay therefore, 9 times 27 is 243, out of 540, and the remain is 297, which being the laſt remain, and greater then the Diviſor, ſhews the quotient 259 is too little.

Wherefore divide the laſt remain 297, By the Diviſor 27, ſaying, twenty ſeven is in 29 once, and write 1 in the new quotient; and ſay once 27 is 27, out of 29, remains 2, which write over 9, and remove the Diviſor, and ſay, 27 in 27, juſtly once, ſo write 1 in the quotient, ſo the quotient is 11, which added to the former quotient 259, gives 270, which is the true and whole quotient.

~~2~~
~~297~~ (11
~~277~~
~~2~~

A Second way of Diviſion.

Although (as I have ſaid) the former way is more uſed, yet this may ſeem plainer and more natural to ſome, I will therefore give one example of it.

Example.

Example.

Let it be required to divide 6477734 *by* 334, where the firſt figure in the quotient is eaſily ſeen 1, ſubſtract once 334 out of 647, and write the remain under a line, then ſee how many times 334 in 3137, anſwer 9, by which multiply the Diviſor 334 in another paper, the product is 3006, which ſubſtract out of the remain, then the new remain is 131734.

Again, how many times 334 in 1317, anſwer is 4, for the third figure in the quotient; by which multiply the Diviſor, the product is 1002, which take out of the later remain, as in the margine, then will remain 31534.

6477734 (19494	
334	
3137734	*Rem.*
3006	*Subſt.*
0131734	*Rem.*
01002	*Subſt.*
0031534	*Rem.*
003006	*Subſt.*
0001474	*Rem.*
0001336	*Subſt.*
. . . . 138	*Rem.*

Now try how many times 334 in 3153, anſwer is 9, for the fourth figure in the quotient, by which multiply the Diviſor (in a by-paper) the product is 3006 (as might have been ſeen above in the ſecond ſubſtraction) which ſubſtracted out of the later remain, there remains now 1474.

Laſtly, ask how many times 334 in 1474, anſwer

ſwer is 4, for the laſt figure in the quotient, by which multiply the Diviſor 334, the product is 1336, which ſubſtracted from 1474, there finally remains 138, which being leſs then the Diviſor, ſhews the diviſion is done.

Proof of this.

	334....
	3006...
The ſeveral ſubſtractions and the final remain added together.	.1002..
	..3006.
	...1336
	138
The Total equal to the dividend	6477734

If the former (as I have ſaid more uſual) way ſeem difficult to Beginners, becauſe the products of the diviſor into the ſeveral figures in the quotient are not ſet down, but mentally made; and alſo becauſe the ſubſtraction of them begins towards the left hand: and laſtly, becauſe the remain is ſtill ſet above: yet this later way which agrees altogether with plain Subſtraction before taught, I hope is ſo plain, that any diligent Reader may acquire it, without a Tutor. And yet for the better ſatisfaction and help of the young Learner, I will add another way or two more of Diviſion.

A Third way of Division.

There is another kind of Division which is very much used, and is in great request with those who have most occasion to divide great numbers, the manner of working is not much unlike the way before taught, one or two *Examples* will make it plain.

Example 1. *Let it be required to divide* 162483 *by* 1321. Set down your numbers as you see them placed in the margine, *viz.* First, set down 162483 the dividend, then on the left hand thereof set the divisor 1321 with a crooked line between them, then on the right hand thereof make another crooked line which must serve to set the figures of the quotient in, so are your numbers placed in due order, then draw a line under the dividend, and make a prick under the figure 4, (because so far the figures of the divisor would extend if they had been placed underneath the dividend, according as in the other examples) this prick serves onely to shew how far you have proceeded in your Work, and must at every division be removed a place further, till at length you come to the last figure of the dividend: your numbers being thus placed with a line under them, you are ready for the Work, which must be performed according to the directions of the following Rule.

1321) 162483 (

THE

THE RULE

Demand how often the Diviſor *may be had in the* Dividend, *and place that number in the* quotient; *then multiply the* diviſor *by the* quotient, *and place the* product *under the line: then ſubſtract this* product *from the* dividend, *and ſet the* remainder *under the* product, *then make a prick under the next figure of the* dividend, *and bring that figure down to the* remainder, *and then proceed as before.*

Example, Your numbers being placed as is before directed, you may begin your Work in this manner; firſt, ſay how many times 1321 can I have in 1624? ſay once, place 1 in the quotient, by which I multiply the diviſor 1321, beginning at the left hand, ſaying, once one is 1, place 1 under the line, then once 2 is 2, ſet 2 under the line, then once 3 is 3, place 3 under the line, laſtly, once 1 is 1, place 1 under the line; then ſubſtract this 1321 from 1624, and there will remain 303. To this 303 bring down the next figure in the dividend, namely 8; (firſt, making a prick under the 8) ſo will that number be 3038, under which draw a line, and repeat the ſame Work again, ſay-

```
1321) 162483 (123
         ...
      ------
      1321
       3038
      ------
       2642
       3963
      ------
       3963
       0000
      ------
```

ſaying, how many times 1321 can I have in 3038, which may be had two times, place 2 in the quotient, by which 2 multiply the diviſor 1321; ſaying, 2 times 1 is 2, place 2 under the line: then 2 times 2 is 4, place 4 under the line: then 2 times 3 is 6, place 6 under the line; laſtly, 2 times 1 is 2, place 2 under the line; and ſubſtract this 2642 from 3038, and there will remain 396, to this 396 bring down the next figure of the dividend, which is 3, ſo is this number made 3963, under which draw a line, and repeat the work once again; ſaying, how many times 1321 can I have in 3963, which may be had 3 times, by which 3 multiply the diviſor 1321; ſaying, 3 times 1 is 3; then 3 times 2 is 6; then 3 times 3 is 9, and laſtly, 3 times 1 is 3, which place under the line: and ſubſtract it from the line above, which in this example is the ſame number, therefore there remains nothing, and the work is ended, but if any remainder had been, that ſhould have been ſet under the line, as by the examples following will appear.

Other Examples for Practice.

```
5624) 793058 (141
        . . .
      ------
      5624
      23065
      -----
      22496
        5698
      ------
        5624
          74 remain-
             der.
```

In this Example where 793058, is divided by 5624, you may perceive that the quotient is 141, and 74 remaining, ſo that the real quotient is 141 $\frac{74}{5624}$.

The

The Proof of this Division.

This kind of diviſion is proved by Addition, for *If you add the ſeveral* products *ariſing from the multiplication of the ſeveral* quotients *into the diviſor, and alſo add thereunto the* remainder (*if any be*) *the total of this addition ſhall be equal to the dividend, if there be no error in the work.*

So in the example following, if you add 4325 the firſt product, and 30275 the ſecond product, and 2796 the remainder together, in the ſame order as they now ſtand in the example, you ſhall find the Total of this Addition to be 76321, equal to the dividend, which demonſtrates the Work to be true.

```
              4325)76321(17
                      ..
                  -------
   1 Product      4325
                  33071
                  -------
   2 Product      30275
                   2796 remain.
                  -------
                  76321 Proof.
```

A Fourth way of Diviſion.

There is a fourth way of Diviſion uſed by ſome, not inferior to any of the preceding, for that it is no burthen to the memory, and it is alſo proved by Addition.

The manner of placing the figures is the ſame with

with the third kind of division last taught; And for the performance of the Work, this is

THE RULE.

First write down the dividend, *and on the left hand thereof the* divisor, *with a crooked line betwixt them, and on the right hand of the* dividend *make another crooked line wherein to place the figures of the* quotient, *then draw a line under the* dividend, *and also make a prick under that figure of the* dividend *under which the last figure of the* divisor *would fall, were it to be placed as in the first kind of* division. *This done, demand how often the* divisor *may be found in those figures of the* dividend, *and place that digit in the* quotient, *then by this digit multiply the* divisor, *and set the* product *of this Multiplication directly under the* dividend, *beginning at the place where you made the prick, then substract this* product *from the figures of the dividend, and place the* remainder *over the* dividend, *cancelling the figures of the* dividend *as you proceed, so is the first figure of the* quotient *finished, then make a prick under the next figure of the* dividend, *and demand how often the* divisor *may be found in the last* remainder, *and the other figure being added thereto, which place in the* quotient, *and proceed in all respects as before, till you have pointed all the figures of the* dividend.

Example, *Let it be required to divide* 763258 *by* 2345. Place your numbers as you see in the margine, and because there are 4 figures in the divisor, therefore make a prick under the fourth figure of

2345) 763258 (

of the dividend, which is 2, and draw a line, then begin your division in this manner: saying,

First, how many times 2345 can I have in 7632, (or how many times 2 can I have in 7) say 3 times, place 3 in the quotient, by which 3 multiply the divisor, saying 3 times 5 is 15, place 5 under the prick,

```
      5 9 7
2345) 7 6 3 2 5 8 (3
            .
     ----------
      7 0 3 5
```

then 3 times 4 is 12, and 1 is 13, place 3 under the line; then 3 times 3 is 9, and 1 is 10, place a cypher under the line; then 3 times 2 is 6, and 1 is 7, place 7 under the line; then substract 7035 from 7632. saying 5 from 12, and there remains 7, place 7 over 2, (cancelling the 5 and the 2) and bear one in mind, then 1 and 3 is 4, out of 13, there remains 9, place 9 over 3, and cancel 3 and 3, then 1 which I carried from 6, and there remains 5, place 5 over 6, and cancel 0 and 6; lastly, 7 from 7 there remains 0, which you need not set down, but cancel the two sevens, then will the Work stand as above, and the remainder will be 597.

Secondly, make a prick under the next figure of the dividend, namely 5, and say how many times 2245 can I have in 5975, answer two times, place 2 in the quotient, by which multiply the divisor, saying 2 times 5 is 10, place 0 under 5, and carry 1, then 2 times 4 is 8 and 1 is 9, place 9 under 7; then 2 times 3 is 6, place 6 under 9; lastly, 2 times 2 is 4, place 4 under 5; so is the product of this multiplication 4690, which you must substract from 5975, saying 0 from 5 and there remains 5, place

5 over 5, and cancel 0 and 5; then 9 out of 17, there remains 8, place 8 over 7, and cancel 9 and 7, then 1 carryed and 6 is 7, from 9 there remains 2, place 2 over 9, and cancel 6 and 9, laſtly, 4 from 5 reſts 1, place 1 over 5, and cancel 4 and 5, ſo have you finiſhed your ſecond figure, and your work will ſtand thus, and your remainder will be 128.

```
       1 2 8 1
       5 9 7 5
2345)7 6 3 2 5 8(32
           . .
      ---------
      7 0 3 5 0
       4 9 6
```

Thirdly, make a prick under the next figure of your dividend (namely under 8) and ask how many times 2345 can I have in 12858 (or how many times 2 can I have in 12) ſay 5 times, place 5 in the quotient, by which multiply the diviſor, ſaying 5 times 5 is 25, place 5 under 8, and carry 2; then 5 times 4 is 20, and 2 is 22, place 2 under 0, and carry 2; then 5 times 3 is 15, and 2 is 17, place 7 under 9, and carry 1; then 5 times 2 is 10, and 1 is 11, place 11 under 4 and 6; ſo is the product of this multiplication 11725 to be ſubſtracted from 12858, ſaying 5 from 8 reſts 3, place 3 over 8, and cancel 5 and 8; then 2 from 5 reſts 3, place 3 over 5, and cancel 2 and 5; then 7 from 8 reſts 1, place 1 over 8, and cancel 7 and 8; then 1 from 2 reſts 1, place 1 over 2, and cancel 1 and 2; laſtly, 1 from 1 reſts nothing, ſo is your work ended, which you ſhall find to ſtand as in the margine, the remainder being 1133.

```
          1 1
        1 2 8 3
        5 9 7 5 3
2325) 7 6 3 2 5 8(325
            . .
      -------------
      7 0 3 5 0 5
       4 6 9 2
       1 1 7
```

I had an intent here to have put an end to *Division*, But there came into my mind

A Rule, by which you may certainly know, what figure to ſet in your Quotient, and never to take one too great, or too little; but that which will juſtly ſerve: And alſo to perform (with eaſe and certainty) the hardeſt, and moſt difficult Sum, that can be propoſed in *Diviſion*, without the aſſiſtance of *Multiplication*, only by *Addition* and *Subſtraction*, not burthening the memory at all.

In the practice of *Diviſion*, there is nothing more difficult than in large Sums (eſpecially if the firſt figures of the Diviſor be either 1, 2, 3, or cyphers, and the laſt figures 7, 8 or 9) to know certainly what figure to put in the Quotient, when you demand how often the diviſor may be had in the dividend, for the certain finding whereof (a little pains being taken before you begin your Work) do thus.

Suppoſe you were to divide any Sum, as 1979909, by 309: Firſt, ſet down the nine Digits 1, 2, 3, &c. one under another, and againſt the figure 1, ſet 309 your diviſor, which doubled is 618, which ſet againſt 2, theſe added together make 927, which ſtands againſt 3; Add the diviſor 309 to 927, it makes 1236, which is againſt 4, to this add the diviſor, and it makes 1545, which ſtands againſt 5. And thus to every laſt number,

1	309
2	618
3	927
4	1236
5	1545
6	1854
7	2163
8	2472
9	2781

number, ſtill add the *Diviſor* till you have gone through all the nine digits, then will they be as in the margine,

Having prepared this little Table, ſet your dividend and diviſor down, as in the Third way of Diviſion pricking the dividend, and drawing a line under it, as is there directed, and as you ſee here done.

```
309)1097909(3553
        ....
   ------------
    927
   1709
   ------------
   1545
    1640
   ------------
    1545
      959
   ------------
      927
       32
```

Then laying your little Table before you, look in it for 1097, the four firſt figures of the dividend, which you cannot exactly find there, but the neareſt number leſs (which you muſt alwayes take when you cannot find the juſt number you look for) is 927, againſt which ſtands 3, ſet 3 in the Quotient, and ſubſtract 927 out of 1097, and there will remain 170, to which bring down 9, the next figure of your dividend, and it is 1709. Look this number in your Table, which you cannot find, but the

next

next lesse is 1545, against which stands 5, set 5 in the Quotient, and substracting 1545 from 1709, there will remain 164, to which bring down the next figure of your dividend (which here is a cypher) making it 1640: Look this 1640 in the Table, which you cannot find, but the next lesse is 1545, against which stands 5, set 5 in the Quotient, and substract 1545 out of 1640, and there will remain 95; to which bring down the last figure of your dividend 9, making it 959. Look this number in the Table, or the next lesse, which is 927, against which stands 3, set 3 in the Quotient, and substract 927 from 959, the remainder is 32. So is your division ended, and the Quotient is $3553\frac{32}{309}$. And with what ease and certainty this is effected; no Multiplication being used, I leave to the Reader to judge.

The Proof of this Division.

This kind of division is also proved by Addition; for, *If you draw a line under the Work, and add all the figures between the two lines together, (in order as they there stand) taking in the remainder (if any be) the Total of this addition will be equal to the Dividend, if the Work be true.*

Other Examples for Practice proved.

```
                (4
              ~~2~~~~2~~~~1~~ (73
    542) ~~7~~~~6~~~~3~~~~5~~~~3~~ (140
           · · ·
       ___________
          ~~5~~~~4~~~~2~~8~~0~~
          ~~2~~~~1~~~~6~~
       ___________
          7 5 8 8 0
```

To which add 473 *the remain.*

the ſumme is 76353 *equal to the dividen.*

```
              6
           ~~1~~~~7~~~~6~~~~6~~
           ~~7~~~~4~~~~4~~~~9~~ 4
  5678) ~~2~~~~3~~~~4~~~~5~~~~6~~~~7~~8 (413
               · · ·
        _______________
        ~~2~~~~2~~~~7~~~~1~~~~2~~~~1~~~~4~~
          ~~5~~~~6~~~~7~~~~3~~
           ~~1~~~~7~~~~0~~
        _______________
        2 3 4 5 0 1 4
```

to which add 664 *the remainder.*

the total is 2345678 *equal to the dividend.*

Queſtions

Questions performed by Division only.

Question 1. *If a piece of Land lying in a long square or Parallelogram, contain* 42952 *square perches, and one of the sides thereof be* 236 *perches long, how long must the other side be?* Divide 42952 by 236, the quotient will be 182, and so many perches long must the other side be.

Question 2. *In a year there are* 8760 *hours, and in every natural day there are* 24 *hours, I demand how many dayes be there in a year?* Divide 8760 by 24, the Quotient will be 365, and so many dayes be there in a year.

Question 3. *The distance from* London *to* Coventry *is* 133760 *yards, and in one mile there is contained* 1760 *yards, now I would know how many miles it is from* London *to* Coventry; Divide 133760 by 1760, the quotient will be 76, and so many miles is it from *London* to *Coventry.*

These Questions performed by *Division* only, are the converse of those that were performed by *Multiplication*, which I the rather make choice of, that the Reader might see how *Multiplication* and *Division* prove each other.

There are one or two more kinds of *Division*, something like these last, but I shall forbear exemplifying them; for much variety helps to make a Book rather great then useful.

¶ Here

¶ Here is to be noted, that in the following Rules, where there is continual use of *Division*. I sometimes use one kind of *Division*, and sometimes another, for variety sake, but the Practitioner may use which he is best skill'd in, for they all produce the same effect.

Reduction.

IS two-fold, First, That which turns *greater* denominations into *smaller*, as pounds into shillings or pence, this is done by *Multiplication* as followeth.

Example

Example 1.

Let it be asked how many pence are contained in 719 li. 11 s. 7 d?

First, a ſhilling is contained in a pound 20 times, therefore multiply 719 by 20, or (which is the ſame, but ſhorter) by 2, and put 0 to the product, as in the margine, this ſhews, that in 729 *l.* there are 14580 ſhillings. To which add 11 *s.* it makes 14591 ſhillings. Again, becauſe one penny is contained in one ſhilling 12 times, multiply 14591 by 12, it produceth 175092, to which add the 7 pence, ſo the ſumme will be 175099, and ſo many pence are contained in 729 *li.* 11 *s.* 7 *d.*

li.	
729	
20	*multiply*
14580	
11	*add*
14591	
12	*multiply*
29182	
14591	
175092	
7	*add*
175099	

Example 2.

Let it be asked how many pints are contained in 4 Tuns, 3 Hogſheads, and 27 Gallons?

Firſt, 1 Tun is equal to 4 Hogſheads, therefore 4 Tun is equal to 16 Hogſheads, to which add the 3 Hogſheads, ſo there is 19 intire Hogſheads.

Again

Again, because one Hogshead contains 63 Gallons, multiply 19 by 63, it produceth 1197 Gallons, to which add 27, it gives 1224 Gallons.

Lastly, because every Gallon contains 8 pints, multiply 1224 by 8, it produceth 9792, and so many pints are contained in 4 Tuns, 3 Hogsheads, and 27 Gallons.

```
  63
  19 multiply
 ----
 567
 63
 ----
1197
  27 add
 ----
1224
   8 multiply
 ----
9792
```

After the same sort might dry Measures be reduced, as quarters to bushels, pecks or gallons, and likewise all weights and Outlandish Coins, of which the proportion of the greater to the lesser is (before) known or given.

Secondly, It is often requisite to turn *smaller* denominations to *greater*: this is done by *Division*, as followeth.

Example 1.

Let it be asked how many pounds are contained in 80976 shillings?

Divide 80976 by 20, the quotient is 4048 *l.* and 16 *s.* remaining, which is the true answer.

```
 x (1
8097 (6 (4048
22220
```

Example

Example 2.

Let it be asked how many pounds are in 109754 *d.*?

Becauſe a pound contains a ſhilling 20 times; and a ſhilling contains a peny 12 times, therefore if 109754 be divided firſt by 12, the quotient ſhall be 9146 ſhillings and 2 pence over; then if 9146 be divided by 20, the quotient is 457 pounds and ſix ſhillings remaining; ſo that 109754 pence is equal to 457 *li.* 6 *s.* 2 *d.*

Or if 109754 had been at firſt divided by 12 times 20, that is by 240, (which is the number of pence contained in a pound) the quotient had been 457 pounds, and 74 pence remaining, which is all one with the former; for 74 pence is equal to 6 ſhillings 2 pence.

More inſtances ſhall not need herein, becauſe the thing of it ſelf is very clear.

Pro-

Progression.

IS also of two sorts, the first is of certain numbers in *Arithmetical Proportion* from 1, that is, such as differ equally, as 1, 2, 3, 4, 5, 6, where the common difference is 1, (as is easily seen,) or 1, 3, 5, 7, 9, 11, where the common difference is 2, or any other, as 1, 8, 15, 22, 29, 36, where the common difference is 7, this is called *Arithmetical Progression*.

2. Secondly, of certain numbers in *Geometrical* proportion from 1, that is such as increase by a common Multiplication, as 1, 3, 4, 8, 16, 32, where the common Multiplier is 2, that is the first by 2, produceth the second, and the second multiplied by 2, produceth the third, and so on.

Or as, 3, 9, 27, 81, 243, where the common Multiplier is 3, this is called *Geometrical Progression*.

Both the common difference (in the first) and the common Multiplication (in the later) shall for shortnesse hereafter be called the *common excesse*.

First, now of the first sort, or *Arithmetical Progression*, the principal use of this is,

1. *If the number of place, and common excesse be given, to find the last number.*

2. *When the number of places, and the last number*

ber is given, to find the aggregate, or total sum of all the numbers.

3. *When the last number, and the total sum is given, to find the number of places.*

4. *The number of places, and total sum being given, to find the last number.*

5. *The last number, and number of places given, to find the common excesse.*

6. *the last number, and common excesse being given; to find the number of places.*

I will instance in no more, few of these ever happing to be used.

For the first of these, let there be given the number of places 100
The common excesse 1
To find the last number also; 100

THE RULE.

Multiply the number of places lesse by 1, by the common excesse, and to the product add the first number; the sum is equal to the last number.

So here, multiply 99 by 1, the Product is 99, (for 1 neither multiplies nor divides) to this add the first number 1, it gives 100 for the last number.

Or let the numbers be 1, 7, 13, 19, 25, 31, where the common excesse is 6, and the number of places also 6.

Now, if the number of places lesse by 1, that is 5, be multiplied by the common excesse, which is 6, the product is 30, to which adding the first number which is 1, the last number 31, is thereby composed. This is so easie that it needs no proof.

2. For the second, which is, The last number, and the number of places given, to find the total sum of all the numbers.

THE RULE.

Add the first and last numbers together, and multiply the sum by half the number of places, the product is equal to the aggregate or sum of all the numbers added together.

So if to the first number 1 be added the last number 100, it gives 101, which multiplied by 50 (which is half the number of places) produceth 5050, which is equal to all the hundred numbers added together.

And hereby may that vulgar question be answered, which is,

If a man take up 100 stones placed a yard one from another, all in a right line, by one at once, and bring them back one by one to his first standing, how many yards doth he go backwards and forwards?

It is shewed before that he goes forward 5050 yards, and he must needs come back just as much; that is, in all 10100 yards, which is 5 miles and 3 quarters; wanting 20 yards.

Or secondly, suppose the numbers were 1, 9, 17, 25, 33, 41. Whereof the common excesse is 8, the first and last added gives 42, which multiplied by 3, (half the number of places) the Product is 126 which is the sum of them all.

3. For the third thing, that is, by the last number and the total, to find the number of places.

THE

THE RULE

Add the first and last numbers, and by the sum of them divide the total, the quotient will be equal to half the number of places.

x x 6 (3
4 x

This is so plain it needs no cleering.

4. For the fourth, if the total, and number of places be given, to find the last number.

THE RULE.

Divide the total by half the number of places, the quotient is a number, from which if 1 *be taken, the rest is the last number.*

As let the numbers be 1, 3, 5, 7, 9, 11, 13, 15, or any other (in Arithmetical proportion) whatsoever. The sum of these is to be 64. And the number of places is 8, the half of it 4. Now if 64 be divided by 4, the quotient is 16, from which if 1 be taken, there remains 15 for the last number.

x 0
64 (16
44

5. Now for the fifth variety, If the last number and number of places be given, to find the common excesse.

THE RULE.

From the last number take 1, *and the remain shall be the Dividend; then from the number of places also take* 1; *and make this later remain the Divisor; then the quotient of this Division shall be the common excesse.*

Example. Let the numbers be 1, 4, 7, 10, 13, 16, from 16 take 1, remains 15, for the dividend, then from 6, (which is the number of places) take alſo 1, remains 5 for the Diviſor.

Now when 15 is divided by 5, the quotient is 3. And 3 is alſo the common exceſſe, or difference between 1 and 4, or 4 and 7, &c.

6. Laſtly, let the laſt number, and the common exceſſe be given, to find the number of places.

THE RULE.

From the laſt number take 1, *and divide the remain by the common exceſſe; then to the quotient add* 1, *the ſum is the number of places.*

As, let the numbers be 1, 5, 9, 13, 17, 21, 25, 29, from 29 take 1, remains 28, which divided by 4 (which is the exceſſe) the quotient is 7, to which add 1, the ſum is 8, which is the number of places, as the reader may eaſily count.

Geometrical Progreſſion.

I ſhall not be ſo large in this as in the former, becauſe theſe things are of little uſe to the Arithmetician, except where a number is to be many times doubled, tripled, or the like, which cannot be ſo eaſily abridged here, as in the other, becauſe there the laſt number ariſing of many *Additions* of the exceſſe to 1, was eaſily found by one multiplication: but here the laſt number being made by many *Multiplications* of the exceſſe, is therefore many times

harder

harder then the other.

The varieties here ſhall be but two.

1. *The common exceſs and number of places being given, to find the laſt number.*

2. *The exceſſe, and laſt number being given, to find the total ſum.*

The firſt of theſe may thus be found. Let the numbers be 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, the exceſſe is 2, the places 10, find out the fifth number (which is eaſily done, for any one may reckon ſo far by heart, that is here 16, and multiply 16 by 16, it produceth 256, which is the ninth number, laſtly, multiply 256 by the exceſſe 2, thence ariſeth 512, the number deſired.

So if the places had been more, as 72, having found the 9 number 256, multiply it by 256, thence comes 65536 for the 17th. number, which multiplied by the exceſſe 2, gives 131072 for the 18th. place, which multiplied by 131072, gives 17179869184, for the 35 place; and that multipled again by the exceſſe 2, gives 34359738368, for the 36 place, that multipled by 34359738368, the product will be 1180591620717411327424, for the 71 place, which laſtly, multiplied by the exceſſe, gives 2361183241434822654848, for the 72 place, which is the laſt number of the Progreſſion required to be found.

Perhaps this may ſeem ſomewhat tedious, but where things cannot be performed without labour, the Reader muſt content himſelf with ſuch Rules as make it leſſe; for it is certain that this way is much ſhorter then to have multiplied ſtill by the exceſſe 71 times, which elſe he muſt have done.

All this notwithstanding, he is not bound to use the same numbers, much lesse in other questions where the number of places is not the same, but whereas I began from the place 9, he may begin at 8, 10 or 12, or where he pleases; so as he remembers still where he is; for this is general, *If the number belonging to any place whatsoever, be multiplied by it self, the product shall be the number belonging to twice so many places want one place.*

Now for the second thing, which is to find the sum of all the numbers.

THE RULE.

From the last number take the first, and divide the remain by the excesse want 1, *then multiply the quotient by the excesse; and to the product add the first number, the sum of them is equal to the sum of all the numbers.*

So if from the last number, or 72 place be taken 1, remain is 2361183241434822654847 which should be divided by 1, (that is the excesse want 1 for the excesse is but 2) but because 1 neither multiplies nor divides, that labour is saved; Now multiply this remain by the excesse, the Product is 4722366482869645309696, to which adding the first number 1, by making the figure 6 next the right hand to be 7, you have the total sum of all the 72 numbers.

A Question resolved by Geometrical Progression.

A Londoner *sojourning in a Countrey Market Town, in Winter made himself a new Freez Suit and Coat, on which were set 6 dozen of Buttons of Silk and Silver; A Baker being in his company liked it so well he would buy it of him; the Citizen consented to let him have it, paying for the first Button a single Barley Corn, for the second 2, for the third 4, and so on doubling to the last.*

The bargain was liked on both parts for the present, but shortly after revoked, for it could not be performed, and no man can be holden to an impossibility.

But why this could not be performed: may be judged; First, by inquiring the worth of so much Barley in Money: And secondly, the weight of it; and how it should be removed,

1, For the first, allowing 10000 corns to a pint (which is more then enough) then 5120000 corns make a quarter; and yet (for shortning the Division) we will allow 10000000 corns to a quarter; by which dividing the whole number of corns (which is done by cutting off the first 7 figures towards the right hand,) the quotient will be 472236-648286964, and so many whole quarters there are omitting the remain, as in this case considerable.

Now allowing Barley were to be sold at 15 *d.* the Bushel (which is cheap;) it is so many Angels: and therefore dividing by 2 it is 236118324143-482 pounds sterling: which is in words, *Two hundred thirty six millions of millions, one hundred and*

eighteen thousand, three hundred twenty four millions one hundred forty three thousand, four hundred eighty two pounds, which I take to be too much for any Trades-man to get or keep.

And reckoning land for ever at twenty years purchase, if this sum of pounds be divided by 20, the quotient is the yearly rent of 11805916207174 pounds.

And this divided again by 365 (the number of dayes in a yeer) the quotient 32344975918, that is above *Thirty two thousands of millions a day for ever.* So great a vanity may be concluded on for want of a little premeditation.

2. Now secondly, for the weight of it, if we put 8 Bushels to weigh 2 hundred pound weight, (for sure it doth weigh more) then the whole number of quarters multiplied by 2, gives the weight of all to be 944473296573928 hundred weight, and if this be divided by 20, (which is but cutting of one figure toward the right hand, and dividing the rest by 2) or which is all one, cut off one figure from the number of quarters, the quotient 47223664828696 is so many tuns. And therefore it will require 47223664828 ships of 1000 a piece to carry it: And consequently, if every Nation in the World had above 10000 such ships, yet there must be above *four millions* of such Nations: which I suppose are not to be found in this World.

And here I will leave this, having used this long example, (which though it require more labour as all great examples do; yet the same skill will do it, as if the places had been fewer) that the Reader being throughly exercised thereby, may the easier leap over others which are shorter.

THE

THE GOLDEN RULE, Or, Rule of Three Direct.

THis is the most *useful* and most *easie Rule* in *Arithmetique*, and deserves a *Golden name*. It is when there are three numbers given, or known, to find a fourth in proportion with them.

But 4 numbers are in proportion, and called *Proportional*, when as the first is to the third, so is the second to the fourth.

As if there were given, 3, 4, and 6, to find a fourth, which may be to 4, as 6 to 3, that is double, and that fourth number is 8; And this is called *Proportion direct*: and the Rule whereby it is done, *The Direct Rule*.

There is also another proportion called *Reciprocal*; which is when as the first is to the third, so is the fourth to the second: As 3, 4, 6, and 2, this is called *The Reverse Rule*.

In *Direct proportion*, the product of the two middle numbers multiplied together, is ever equal to the

the product of the first and last multiplied together, which serves not only for a *Proof*, but a ground of the *Rule*, which *Rule* shall here follow: the *Reverse Rule* being deferred till we have done with this.

The Rule Direct.

Multiply the second term (or number) by the third, and divide the Product *by the first; the* Quotient *shall be the fourth number desired.*

Example. Let the three numbers given be 2, *6*, 3, multiply *6* by 3, the Product is 18, then divide 18 by 2, the Quotient is *9*, which is the fourth number in proportion with 2, *6*, and 3.

For as 2 to 3, so 3 times 2, which is *6*, is to 3 times 3, which is *9*.

And so the product 18 divided by 2, and the quotient *9*, causeth that the product of 2 into *9* shall be also 18, and consequently if 2 be the first of the 4 proportional numbers, and *6* and 3 the two middlemost, then *9* is the last.

Otherwise.

Divide the second by the first, and multiply the third by the quotient, the product shall be the fourth.

So if one divide *6* by 2, the quotient is 3, by which multiply 3, the product is *9*, for the fourth number, as before. Otherwaies this Rule might be

be expreſſed; but where the firſt way is ſo ſhort and clear, there many other wayes would rather trouble then help the perſon that ſhould uſe them.

In the firſt way (which here we mean to uſe and no other) if the *firſt* number be 1; then the product of the *ſecond* and *third* gives the *fourth*, without any *Diviſion*. Or, if the *ſecond*, or *third*, number be 1, then there needs no *Multiplication*, but dividing the greater of them by the firſt, the quotient (in whole numbers for yet we ſpeak of them) is the fourth number, which was ſought.

Note 1.

To know when to uſe the Direct, or the Reverſe Rule. Conſider, if *more*, require *more*; or if *leſſe*, require ſtill *leſſe*: then uſe the *Direct Rule.* But if *more* require *leſſe*, or *leſſe more*, then uſe the *Reverſe Rule*, this will be eaſily underſtood when we come to Examples.

Note 2.

To know how to place the three numbers when they are confuſedly given. Remember that 2 of them are alwayes of one denomination, as both *pounds*, or both *ſheep*, or both *yards*, or *acres*; and the other number hath another denomination; now know, that this ſingle number is ever the ſecond number in order.

And

And one of the other two, namely, that which hath ſome relation to this *ſecond*, is the *firſt*; and the other is the *third* number, whoſe relation is ſought for in the *fourth*, whence its plain that the *ſecond* and *fourth* are alſo of the ſame denomination.

And having premiſed theſe things, let us now exemplifie the Rule in ſome queſtions.

Queſtion 1.

If 3 *yards of cloth coſt* 4 li. *what ſhall* 21 *yards coſt?*

Set the numbers in order as in the example. If 3 yards coſt 4 *li.* what 21 yards? Here you ſee that the firſt number and the third number are both of one denomination, *viz.* both *yards*, and the ſecond number is of another denomination, namely *pounds*, wherefore the *fourth* number which is ſought for, muſt be alſo *pounds*, therefore multiplying (according to the Rule before given) the *ſecond* number by the *third*, and dividing the Product by the *firſt*, the quotient ſhall anſwer the queſtion.

Firſt, 21 multiplied by 4, (which is the third number multiplied by the ſecond) produceth 84, which divided by 3 the firſt number, the quotient is 28 *li.* and ſo much ſhall 21 yards coſt: for 28 is to 4, as 21 to 3, ſeeing each contains either 7 times.

And

And the work will ſtand thus,

yards *pounds* *yards*
If 3 coſt 4 what 21
4
x *li.*
~~8~~~~4~~ (28 84
~~3~~~~3~~

Queſtion 2.

If 4 *men eat* 2 *pecks of Corn in a week, how many pecks ſhall ſerve* 100 *men?*

Place your numbers as here you ſee, then multiply 100 by 2, (that is the third number by the ſecond, and the Product is 200, which divided by 4, the quotient is 50, for the number of pecks required.

men *pecks* *men*
If 4 eat 2 what 100
2
200

~~2~~~~0~~~~0~~ (50 *pecks*
~~4~~~~4~~

Queſtion 3.

If 20 *ſheep coſt* 13 *pound* 13 *ſhillings* 4 *pence, what is that for every ſheep?*

Turn

Turn the ſhillings and pounds into pence, thus,

s.	*li.*
13	13
12	240
26	520
13	26
156	3120

3120
156
4

3280

Multiply 12 *s.* by 12, the product is	156
And 13 *li.* by 240 (becauſe 240 pence make one pound) the product is	3120
To which add the 4 *d.*	4
It makes in all	3280

Then the queſtion will be, If 20 ſheep coſt 3280 pence, what ſhall one ſheep coſt?

ſheep *pence*
If 20 coſt 3280 what 1?

x d.
3280(164
2220

x d.
4(8 *s.*
x64(13
xxx
x

By

By the Rule before delivered, I ſhould multiply the ſecond number by the third, but in this example, the third number being 1, it doth not multiply; I therefore divide 3280 the ſecond number, by 20 the firſt number, and the quotient 164, is the price of one ſheep in pence, which divided by 12, the quotient is 13 *s.* and 8 *d.* remaining, the price of every ſheep therefore is 13 *s.* 8 *d.*

Queſtion 4.

How many 10 *inch tiles will pave a floor that contains* 16 *ſquare yards?*

Firſt remember there are 36 inches in one yard in length; which multiplied into 36, gives 1296, for the ſquare inches in one ſquare yard; multiply 1296 therefore by 16, thence comes 20736, the ſumme of all the 16 yards in inches.

Secondly, ſeeing every tile is 10 inches in length, and 10 in bredth, multiply 10 by 10, it produceth 100 for the ſquare inches in one tile, See the manner of work.

```
   36     10
   39     10
 ----   ----
  216    100
 108
 ----
 1296
   16
 ----
 7776
1296
 ----
20736
```

Then by the Golden Rule.

If 100 *inches require* 1 *tile; what shall* 20736 *inches require?*

```
       inches           tile        inches
    If  100  require     1 what  20736.
                         0 |
                       ØØ7 | 36 (207
                       xxx | 00
```

Here because 1 doth neither multiply nor divide (as hath been several times intimated) I therefore divide the third 20736, by the first 100, the quotient is 207, and 36 remaining.

So it appears, that 207 is too little, and 208 too much to do the Work: the just number being $207\frac{36}{100}$, we shall not trouble the Reader with this till he know something of Fractions.

Question

Question 5.

If 100 *li. give* 6 *li. interest for a year, how much shall* 750 *li. give?*

Multiply 750 by 6, the product is 4500, which divided by 100, the quotient is 45 *li.* for the thing required.

li.		*li.*		*li.*
If 100	give	6,	what	750 ?
				6
				4500

li.

4500 (45
1100

Question 6.

If 750 *li. give* 45 *l. interest for a year, what shall* 100 *li. give?*

Multiply 45 by 100, the product is 4500, which divided by 750, the quotient is 6 *li.* for the interest of 100 *li.* for a year.

li. li. li.
If 750 give 45, what 100

100

4500

3
$\cancel{4}\cancel{5}\cancel{0}\cancel{0}$ (6
$\cancel{7}\cancel{5}\cancel{0}$

Many other questions might be added, but the Rule is so plain, that it needs them not; and so general, that he which can resolve one, may aswel resolve any other: And for that reason; and because in all the Rules which follow, this Rule will be constantly made use of, I will say no more of it here.

The Golden Rule *Reverse.*

IF 12 workmen do any piece of work in 8 moneths: how many workmen shall do the same in 2 moneths?

THE

THE RULE.

Multiply the first *term by the ſecond ; and divide the Product by the* third, *the quotient is the number deſired.*

Here 12 is not the firſt number, though it be firſt named ; but the three numbers placed in order, ſtand thus, 8, 12, 2, for the middle term muſt alwayes be of the ſame denomination with that which is required.

Now multiply 12 by 8, the product is *96*, which divided by 2, the quotient is 48, which anſwers the queſtion, As in the example,

moneths	*men*	*moneths*
8	12	2
	8	

2)9*6*(48

8
1*6*

1*6*

For if 8 moneths require 12 men ; then (a fourth part of 8) 2 moneths, ſhall require four times 12, that is) 48 men.

For here *leſs* requires *more* ; that is, *leſs time, more hands* ; and therefore is it wrought by the *Reverſe Rule.*

Question 2.

How many Ells of Tapestry will serve to hang a Room 3 yards high, 6 yards long; and 5 yards broad? not regarding Doors, Windows or Chimney, but as if there were no such.

First, multiply 6 by 3, the product is 18, which doubled (because there are 2 sides called *lengths*) is 36 yards for all the length.

Secondly (for the same reason) multiply 3 by twice 5, that is by 10, the product is 30 yards, for all the bredth; which added to 36, gives 66 yards, equal to all the length and bredth in yards.

But now because Ells, that is, *Flemish* Ells (for such measure are Hangings sold by) is equal to 3 quarters of a yard, that is, their Ell is to our Yard as 3 to 4. Say therefore, if 4 give 66, what 3? multiply 66 by 4, it produceth 264; then divide 264 by 3, the quotient is 88. Again, multiply 88 by 4, and divide the product (which is 352) by 3, the quotient is 117, and 1 remaining, to which the divisor 3 being applyed; the number justly answering the question is 117 Ells, and one third part of an Ell.

Note 1.

Because here we had to deal with things which had equal length and bredth, that is square yards, and square ells; therefore one multiplication and division was not sufficient to proportion this; but if instead of working by 4 and 3, we had done it by their

their squares which are 16 and 9, it might have been performed at once, thus, multiply 66 by 16, the product is 1056, which divided by 9, the quotient is $117\frac{1}{3}$, as before, but I began not with this way, for I supposed my Reader ignorant of squares.

Note 2.

It might also have been done, by reducing all the terms into quarters of a yard at the first, and after the number is found, reducing them again to ells; but because it is more proper to work thus, till fractions have been taught: I leave that, and proceed to another question.

Question 3.

If 1 Close would graze 21 Horses for 6 weeks; then (supposing no waste to be made) how many Horses would it feed for 7 weeks?

Multiply 21 by 6, it produceth 126, which divided by 7, the quotient is 18. At that rate therefore it would keep 18 Horses for 7 weeks.

Question 4.

If 1 Close will feed 18 Horses for 7 weeks, how long shall it feed 63 Horses?

Multiply (according to the rule) 18 by 7, the product is 126, which divided by 63, the quotient is 2, therefore 2 weeks it shall keep them.

The like way serves for Hay, Oates, or any other provision for Man or Beast; which may be of use in

Garrisons, and such like cases where scarcity may be feared, to proportion either the *mouthes* to the *meat*, or *meat* to the *mouthes*.

Before I leave this *Rule*, (because it comes not so much in use and practise as the *direct Rule* doth, and therefore may be more apt to be forgotten) I will, to exercise the Reader therein, propose the following *Questions*, giving the Answers of them, and leave the practice to the Reader to find out of himself, the better to fix it, the Rule in his memory.

Quest. 1.

If 12 *men will raise a Frame in* 10 *dayes; in how many dayes would* 8 *men raise the same?*

Here, because the fewer men would require the longer time, though the numbers be 12, 10, 8, yet you shall (by observing what hath been already delivered in this Rule) find the fourth proportional (which is the number answering the Question) to be 15, and so many men will do the Work in 8 dayes.

Question 2.

If 60 *yards of Hangings of three quarters broad would hang a Room; How many yards of half a yard in bredth would serve to hang the same Room?*

Answer, Ninety yards.

Quest. 3.

If a board being 12 *inches in bredth do require*

12

12 inches in length to make a foot square; What number of inches in length will make a foot square, when the bredth of the board is 16 inches?
Answer 9 Inches.

Question 4.

If the base or end of any solid (as a piece of Timber or Stone) being 144 inches, do require 12 inches in length of that piece to make a solid foot; What number of inches in length will make a solid foot, when the square at the end is 216 Inches?
Answer, 8 inches.

I will say no more of this Rule; Neither will I treat of the *Double Rule of Three*, as a rule by it self; but come to the *Rule of five numbers*, which is an abridgment of the other.

The Golden Rule *Compound of five Numbers.*

Question 1.

I*F a hundred pound weight* (that is 112 pound weight) *carryed 120 miles cost 14 s. how much shall three quarters of a hundred* (that is 84 pound) *cost, being carryed 40 miles?*

THE RULE.

Multiply the three last numbers one into another, (that is) the third *by the* fourth, *and that product by the* fifth; *the last product shall be the* Dividend.

Again, *Multiply the* two first *numbers together; the product shall be the* Divisor. *This Division being made, the* Quotient *will be the number of shillings desired.*

Example of the former Question.

First, place your numbers according to the tenor of the question thus,

li.	*miles.*	*s.*	*li.*	*miles.*
112	120	14	84	40
120		12		
2240		28		
112		14		
13440		168 *pence*		
		84		*d.*
		672	13440)564480(42	
		1344		
		14112	53760	
		40	2688	
		564480	26880	

Your numbers being placed in order, reduce the 14 *s.* into pence, and it is 168 *d.* then multiply

168

168 by 84, the product is 14112, which multiplied by 40 the later product, it produceth 564480 for the dividend.

Then multiply 112 by 120, it produceth 13440 for the Divisor.

Divide 5644800 by 13440, the quotient will be 42 pence; which is 3 *s.* 6 *d.* and answers the question.

In this Rule, the *first* number and *fourth*, also the *second* and *fifth*; and also the *third* and *sixth*, are of like denomination and nature.

Question 2.

If 10 li. *for* 6 *moneths yield* 3 li. *interest, what shall* 625 li. *yield for* 36 *moneths?*

Place them thus, 100, 6, 3, 625, 36.

Multiply the three last, as before is shewed, the later product is 67500 for the *dividend*; And the 2 first multiplied make 600 the *Divisor*, then divide 67500 by 600 (or 675 by 6, which is all one) the quotient will be 112 whole pounds; and 300 (or 3) remaining, which because it is half the divisor, signifies the half of a pound; that is 10 shillings. So the answer to the question is 112 *li.* 10 *s.*

li.	*m.*	*li.*	*li.*	*m.*
100	6	3	625	36
6			3	
600			1875	
			36	
	x3			
6)675(112			11250	
666			5625	
			67500	

Which might have been given in one denomination, namely 2250 ſhillings, if before the work the pounds had been turned into ſhillings, by multiplying them by 20, as hath been ſhewed before.

But ſince moſt queſtions, except ſuch as are ſtudied for the purpoſe, are apt to end in ſome fraction, I ſhall next treat of fractions.

Onely firſt, having ſpoken of the double Rule of Three, this may let you know, that all queſtions which are wrought at once by the compound Rule of Five: may be done at twice by the ſingle Rule of Three; and the doing of them ſo by two operations is called, *The Double Rule.*

As in our laſt queſtion, there are two things conſiderable, the difference of money; and the difference of time.

Firſt, for the money,

Say, if 100 *li.* give 3 *li.* what 625 *li*? anſwer $18\frac{75}{100}$ *li.*

Secondly, for the time;

Say,

Say, if 6 *mo.* give $18\frac{57}{100}$ *li.* what 36 *mo.* anſwer $112\frac{50}{100}$ *li.*

But this will be better underſtood anon; and then the Reader may uſe that which he likes beſt.

Of Fractions.

THe word *Fraction* ſignifies a *breaking* or *breach* of any intire thing into parts; and when a number is broken ſo, the parts (which muſt needs be every one leſſe then the whole; and the whole is accounted but *One* or *Unity*) being leſſe then Unity, are called Fractions (that is fragments or pieces) of Unity. Now the Unite, or intire number which is to be broken, may be any thing, as one *pound*, in reſpect of which, *ſhillings*, and *pence* & *farthings* are *Fractions*; or, one *ſhilling*, in reſpect of which, *pence* and *farthings* are *fractions*; or, one *peny*, in reſpect of which, *farthings* are *Fractions*; and the like of *weights* and *meaſures*, or any other thing to be broken into parts.

In *Fractions*, we ſhall treat firſt of *Numeration*, then of *Multiplication* and *Diviſion*, then of *Reduction*; and laſtly, of *Addition* and *Subſtraction*.

The reaſon of this Order will ſoon be ſeen; for *Multiplication* and *Diviſion* are here much eaſier then *Addition*, &c. and therefore ought to be learned before them.

Nume-

Numeration.

NUmeration is nothing else but the way of writing Fractions; and that this may be done, we must consider that any *Unite*, or *Number* representing an *Unite*, may be broken into two parts equal; and then each of the parts is called *one second*, or *half*; or it may be parted into three equal parts, and then each part is called *one third*; and two of them are called two thirds; and the like may be understood if it were parted into 4, 5, 6, 7, 8, 9, 20, 50, or 100, or how many soever.

Now to write these; do thus,

Write	one half	Thus	$\frac{1}{2}$
	one third		$\frac{1}{3}$
	one fourth		$\frac{1}{4}$
	one fifth		$\frac{1}{5}$
	one sixth		$\frac{1}{6}$
	one seventh		$\frac{1}{7}$
	one eighth		$\frac{1}{8}$
	one ninth		$\frac{1}{9}$
	one tenth		$\frac{1}{10}$

In every one of these 10 *Fractions*, the Number below the line is called the *Denominator*, and it shews into how many parts the *Unite* is broken.

The number above the line shews how many of those parts are taken, or contained in the *Fraction*, and

and is therefore called the *Numerator* : So in the Fraction $\frac{3}{5}$: the *Denominator* 5 ſhews the *Unite* to be broken into 5 parts : and the *Numerator* 3 ſignifies 3 of ſuch parts to be contained in the *Fraction* ; which Fraction therefore is called *three fifths*.

And here it is plain ; that, As the *Numerator* is in proportion to the *Denominator* : ſo is the Fraction to 1, or Unity, for $\frac{3}{3}$ or $\frac{5}{5}$: or any the like, is equal to 1.

And therefore all Fractions are quotients of leſſer numbers divided by greater, as $\frac{4}{7}$ ſignifies 4 to be divided by 7, and as the dividend 4, is to the diviſor 7 : ſo is the quotient $\frac{4}{7}$ to Unity.

And therefore this line of ſeparation which is drawn between the *Dividend* and *Diviſor*, doth properly ſignifie *Diviſion*.

Hitherto we have ſpoken only of ſuch Fractions as are leſs then 1, and thoſe are called *Proper Fractions* : but there are alſo $2\frac{1}{2}$, $3\frac{3}{4}$, $5\frac{1}{7}$, $6\frac{3}{5}$, and the like mixed Numbers ; which ſo written ſignifie *two and an half*, 3 *and* 3 *quarters*, *five and a ſeventh*, 6 *and* 3 *fifths*. Theſe by multiplying the whole Numbers, by the Denominator, and to the product adding the Numerators reſpectively, are turned to $\frac{5}{2}$, $\frac{15}{4}$, $\frac{36}{7}$, $\frac{33}{5}$, which are called *Improper Fractions*, becauſe every one of them contains more then Unity.

Theſe, nevertheleſs may be *multiplyed*, *divided*, *added*, or *ſubſtracted* in the ſame way as are proper Fractions. And this ſhall ſerve for *Numeration of Fractions*.

Mul-

Multiplication.

THE RULE.

M*ultiply all the* Numerators *together, the last product shall be the* Numerator *of the product required: Likewise multiply all the* Denominators *together, the last product shall be the* Denominator *of the product sought.*

Example 1.

If $\frac{3}{5}$ be to be multiplyed by $\frac{4}{9}$, Multiply the *Numerator* 3 by the *Numerator* 4, the product is 12, for the *Numerator* of the new product. Also multiplying the *Denominator* 5, by the *Denominator* 9, they produce 45, for the *Denominator* of the desired product, so that product which was required, is $\frac{12}{45}$.

Example 2.

If $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{3}{11}$ were to be multiplyed all together, begin with the *Numerators*, saying, once 3 is 3, and 3 times 4 is 12, and 12 times 5 is 60, and 60 times 3 is 180, for the *Numerator*: Then multiply the *Denominators*: saying, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 9 is 360, and 360 times 11 is 3960, for the new *Denominator*.

So

So that the product of all these is $\frac{180}{3960}$, that is equal to $\frac{1}{22}$, as shall be seen hereafter in *Reduction*.

And thus it appears, that proper *Fractions* being lesse then *One*, are still made less by *Multiplying*; as here the product $\frac{1}{22}$ is much less then $\frac{3}{11}$, which is the least Multiplyer; and the reason hereof is plain, for seeing *Multiplication* is but the taking of a Number, a certain number of times, if that number of times be more then 1, then the Number to be taken is increased by being taken more then once; but if the Number of times be 1, it is not increased nor diminished, but is still the same; Lastly, If that number of times be less then 1, as $\frac{1}{2}$, the number not being taken once, but half of once, produceth a number lesse by half; that is, the half of the number to be taken; and the like reason is of all others.

Example 3.

Multiply the mixt Numbers $3\frac{1}{2}$, $4\frac{1}{3}$, and $5\frac{3}{4}$: First, (as hath been shewn already) turn them to improper fractions: thus, first, say, 2 times 3 is 6, and 1 is 7. So the first is $\frac{7}{2}$. Secondly, 3 times 4 is 12, and 1 is 13: so the Second is $\frac{13}{3}$. Lastly, 4 times 5 is 20, and 3 is 23: so the last is $\frac{23}{4}$. Now the fractions to be multiplyed are $\frac{7}{2}$, $\frac{13}{3}$, and $\frac{23}{4}$; First, for a new *Numerator*, say, 7 times 13 is 91, and 91 times 23 is 2093, for a new *Numerator*.

Then say, 2 times 3 is 6, and 6 times 4 is 24. So the new *Denominator* is 24.

And the product of all these fractions is $\frac{2093}{24}$, that is, if real division be made, $87\frac{5}{24}$.

Division.

DIvision, To divide one fraction by another, is but the cross multiplication of them; that is, the *Numerator* of the one, by the *Denominator* of the other, and hereby the proportion of one fraction to another is seen.

Example 1.

24

$\frac{3}{4}$ $\frac{6}{8}$

24

Divide $\frac{3}{4}$ by $\frac{6}{8}$, to do it, set them thus: and multiply as the cross leads; Saying, 3 times 8 is 24, which set over the Cross for a new *Numerator*, and 6 times 4 is also 24: which set under the Cross for a new *Denominator*; so the quotient is $\frac{24}{24}$, that is 1, which shews the Fractions to be equal one to another.

Example 2.

27

$\frac{3}{5}$ $\frac{4}{9}$

20

Divide $\frac{3}{5}$ by $\frac{4}{9}$. First, set them thus: And say, 3 times 9 is 27, for a *Numerator*, and 5 times 4 is 20, for the *Denominator*: so the quotient is $\frac{27}{20}$, and so many times is $\frac{4}{9}$, contained in $\frac{3}{5}$, that is, as 27 is to 20, so is $\frac{3}{5}$ to $\frac{4}{9}$, and so is $\frac{27}{20}$ to 1. In

In *Division* it is to be remembred, that the *Numerator* of the *quotient* ever ariseth of the *Numerator* of the *Dividend*; And the *Denominator* of the *quotient* comes of the *Denominator* of the *Dividend*, each being crofs multiplied as before. And alfo remember alwayes to fet the Dividend on the left hand of the Crofs.

If a Fraction be to be divided by a whole number; Multiply the *Denominator* by that *number*, the product gives the new *Denominator*, and the *Numerator* remains the fame. So if $\frac{1}{4}$ be divided by 9, fay 9 times 4 is 36. So the quotient is $\frac{1}{36}$.

Or if $\frac{1}{4}$ were to be multiplyed by 9, the product (by multiplying the Numerator by 9,) will be $\frac{9}{4}$: that is, $2\frac{1}{4}$.

Example 3.

Divide $\frac{320}{8}$ by $\frac{45}{9}$, thus: fay 320 times 9 is 2880, for a *Numerator*: And 8 times 45 is 360 for a Denominator. So the quotient is $\frac{2880}{360}$, or $\frac{8}{1}$.

$$\begin{array}{ccc} & 2880 & \\ \frac{320}{8} & \times & \frac{45}{9} \\ & 360 & \end{array}$$

For $\frac{320}{8}$ is equal to 40, and $\frac{45}{9}$ equal to 5, but 40 contains 5 eight times.

And fo in the fecond example, it may be proved, that as 27 to 20, fo is $\frac{3}{5}$ to $\frac{4}{9}$. For firft, multiply the two middlemoft, then 20 times $\frac{3}{5}$ is $\frac{60}{5}$, that is 12.

Secondly, multiply the firft and laft, and then 27 times $\frac{4}{9}$ is $\frac{108}{9}$: that is alfo 12.

Wherefore by that which hath been faid in the *Golden Rule*, the four Numbers 27, 20, $\frac{3}{5}$, $\frac{4}{9}$, are proportional.

Reduction

Reduction.

REDUCTION OF FRACTIONS is threefold.

1 *To reduce one fraction* (which is not already in the least) *to a lesser Denomination.*

2 *To reduce many fractions of divers denominations, to one denomination.*

3 *To reduce any fraction from one denomination* (as near as may be) *to any other denomination desired.*

I. For the first of these, T*o reduce a* Fraction *to its least* terms. Divide both the *Numerator* and the *Denominator* by the greatest *Common Division* that you can think of; the two *Quotients* being placed respectively in a fraction, that fraction shall be equal to the former fraction, and in lesser terms.

So (in the 3 Example of *Division*) to reduce $\frac{2880}{360}$ to $\frac{8}{1}$, divide 2880 by 360, the *Quotient* is 8: then divide 360 by 360, the *Quotient* is 1, and the new fraction $\frac{8}{1}$ is equal to the former fraction $\frac{2880}{360}$, and in lesse terms, as you may see. But to find the greatest common divisor, this is

The Rule.

Divide the greater term *by the* lesser (I mean by terms,

terms, the *Numerator* and *Denominator*) *and by the remainder* (if any be) *divide the* divisor, *and if any thing still remains, by that divide the last* divisor, *continuing this course till nothing remain greater then* Unity; *that* divisor *which is least of all, is the greatest* Common measure *of both terms, by which both being divided, and the* quotients *placed like a* Fraction, *that* Fraction *shall be equal to the former* Fraction, *and in the* least terms.

Example.

Reduce $\frac{148}{16}$ to the least terms; first divide 148 by 16, the quotient is 9, and 4 remains: again, divide 16 by 4, the quotient is 4, and nothing remains; wherefore taking 4, (the last divisor) for the greatest common divisor, by it divide 148, the the quotient is 37, and by it divide 16, the quotient is 4. These two last quotients placed orderly in a Fraction, make $\frac{37}{4}$, which is equal to $\frac{148}{16}$, and in the least terms, for no number greater then 1, will divide evenly both 37 and 4.

Other wayes there are of lessening Fractions, as dividing the terms (if they be even Numbers) by 2, and the quotients (if even) again by 2, or else by 3, or any other Number that will divide them both evenly, that is, leave nothing remaining, but the former Rule being general and easie, shall serve for all,

II. Now secondly, *To reduce many* Denominations *to one* common Denominator. Let the Fractions be $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, to be reduced all to one denomination.

THE RULE.

Multiply all the Denominators together, and the last Product shall be the common Denominator to all the Fractions———Then multiply every particular Numerator into all the Denominators *except his own, and the last* Product *shall be* Numerator *to that Fraction.*

Thus to reduce the forementioned Fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$ into one Denomination: Say, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 8 is 320, and 320 times 10 is 3200, this last product 3200 shall be the common *Denominator*. Then to get Numerators for every one of them: As first, for the first, say 1 time 4 is 4, and 4 times 5 is 20, and 20 times 8 is 160, and 160 times 10 is 1600. For the first *Numeration*; so the first Fraction reduced is $\frac{1600}{3200}$. Then for the second *Numerator*: Say, 3 times 2 is 6, and 6 times 5 is 30, and 30 times 8 is 240, and 240 times 10 is 2400. So the second fraction reduced, is $\frac{2400}{3200}$. After the same manner may the other three be reduced to $\frac{2580}{3200}$ for the third: $\frac{2800}{3200}$: for the fourth: and $\frac{2880}{3200}$ for the last; these are severally equal to the other, the first to the first, &c. as may be proved thus,

Let the Unity be a pound *Sterling*, then

	s.	d.
The $\frac{1}{5}$ of it is	10	
and $\frac{3}{4}$ is	15	
and $\frac{4}{5}$ is	16	d.
and $\frac{7}{8}$ is	17	6
and $\frac{9}{10}$ is	18	
	s.	d.
In all	76	6

That is 3 whole Unites, and 16 *s.* 6 *d.* over; Turn 16 *s.* 6 *d.* all to six pences, it is 33, and because 6 *d.* is the fortieth part of a pound, therefore all the Fractions are equal to $3\frac{33}{40}$.

Now add the new Fractions (which being all of one denomination) may be added like whole Numbers: thus,

	1600
	2400
	2560
	2800
	2880
In all	12240

Which divided by the Denominator 3200, the quotient is $3\frac{2640}{3200}$. Now $\frac{2640}{3200}$, reduced to the least termes, as hath been shewed how it may, will be $\frac{33}{40}$, so the sum of these also is $3\frac{33}{40}$, which is equal to the sum of the Fractions given to be reduced, and therefore they are equal in sum, and might be thus proved equal severally, that is, the first of them propounded, to the first reduced. Divide the Numerator 1600 by the Numerator 1, the quotient is 1600. Also divide the Denominator 3200, by

 the

the Denominator 2, the quotient is also 1600, and so may any of the rest be proved equal by the equality of quotients. But I leave it as plain enough already.

III. Thirdly, *Any Fraction being given to change the* denomination *to any other more requisite, retaining still* (as near as may be) *the same value.*

THE RULE.

Multiply the Numerator *given, by the* Denominator *required, and divide the* product *by the* Denominator *given*; *the* quotient *shall be the* Numerator *required.*

Example.

Let the Fraction given be $\frac{7}{13}$ of a pound *Sterling*, what is that in the twentieth parts or shillings? Multiply 7 by 20, the product is 140, which divided by 13, the quotient $10\frac{10}{13}$, that is, 10 *s.* and $\frac{10}{13}$ of a shilling: which may be brought to pence thus, multiply 10 by 12, product is 120, which divided by 13, quotient is $9\frac{3}{13}$ *d.* And again, multiply 3 by 4, the product is 12, which divided by 13, quotient is $\frac{12}{13}$ of a farthing, so seven thirteenths of a pound is 10 *s.* 9 *d.* and almost a farthing.

But he which is resolved to have it in the smallest coin, may do it at first work; for seeing a farthing is the 960 part of a pound, multiply 7 by 960, they produce 6720, which divided by 13, the quotient is 516 farthings, and $\frac{12}{13}$ of a farthing: these farthings may be turned to shillings, dividing by 48, or to pence by 4, as in Reduction.

This

This Rule though it be brief and plain; is of great use in Arithmetick; either for turning Natural and surd Fractions into Decimals; or any other desired Denomination, with such facility and speed as may be wished.

IV. *Fraction of Fractions.*

In reduction of Fractions, some make another, or more parts, as *Fractions of Fractions* for one: that is, when there is a part of a *Fraction*, or a part of a part of a *Fraction*, &c. to be valued in one fraction.

THE RULE.

Multiply all the Numerators *together, the last* product *shall be the* Numerator *desired: Then multiply all the* Denominators *together, and this last product shall be the* Denominator *sought.*

Example.

Let the *Fractions of Fractions* propounded, be $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$, for so they are usually written; and let the *Numerators* be multiplyed: saying, 4 times 3 is 12, and 12 times 1 is 12, the *Numerator* therefore required is 12: then for the *Denominator*, say, 5 times 4 is 20, and 20 times 2 is 40, for the *Denominator* required; and $\frac{12}{40}$ is equal to $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$.

Proof.

Let the Unite be 40 *s.* one fifth of 40 is 8, and therefore $\frac{4}{5}$ is 32, of which one fourth is 8, and $\frac{3}{4}$ is 24, of which one half is 12, and therefore $\frac{12}{40}$ is the just sum of all the Fractions, This needs no further exemplifying.

Addition.

TO add many Fractions into one ſum, conſider whether they be all of one Denomination or divers; if of one, Then add all the *Numerators* together into one ſumme, that ſumme is the new Numerator: and the *Denominator*, in this caſe is not altered.

Example.

Let the Fractions to be added be $\frac{2}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{1}{4}$, Add the Numerators: ſaying 2 and 4 is 6, and 5 is 11, and 1 is 12. So the ſumme of them all is $\frac{12}{4}$, that is 3 Unites.

As, let the Unite be 20 *s.* one fourth is 5 *s.* and $\frac{2}{4}$ is 10 *s.* and $\frac{4}{4}$ is 20 *s.* which added to 10 *s.* is 30 *s.* then $\frac{5}{4}$ is 25 *s.* which added to thirty ſhillings gives 55 *s.* And laſtly, $\frac{1}{4}$ is 5 *s.* which added to 55 *s.* makes 60 *s.* that is 3 times 20 *s.* that is 3 *l.* or 3 Unites.

But if the Fractions to be added, be of divers denominations; as let them be $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, then (by the *Reduction* afore-going) they muſt be turned all into one denomination, and then they will be $\frac{320}{480}$, $\frac{360}{480}$, $\frac{384}{480}$ and $\frac{420}{480}$, and may be added like thoſe before: thus,

320

	3 2 0
	3 6 0
	3 8 4
	4 2 0
In all	1 5 8 0

So the summe of all is $\frac{1484}{480}$, or $\frac{371}{120}$, that is, 3 $\frac{11}{120}$, which if it be money, and the Unite 1 *l.* it is then 3 *l.* 1 *s.* and 10 *d.* as may be tryed thus: First $\frac{2}{3}$ of a pound, is 13 *s.* and 4 *d.* and $\frac{3}{4}$ is 15 *s.* and $\frac{4}{5}$ is 16 *s.* Lastly, $\frac{7}{8}$ is 17 *s.* 6 *d.* These all added together, the summe is 3 *l.* 1 *s.* 10 *d.*

Substraction.

IN SUBSTRACTION of one Fraction from another, if they be both of one denomination; It is done by taking the *Numerator* of one from the *Numerator* of the other, the remain is the new *Numerator*, and the *Denominator* the same as before.

So if $\frac{2}{9}$ be substracted from $\frac{8}{9}$, the remain is, $\frac{6}{9}$ the like of all others.

But if they be not of one Denomination, they must first be reduced to be so; then that which is said before is sufficient.

Concerning the Golden Rule in Fractions.

THe *Golden Rule in Fractions* is the same as in whole Numbers, I will give you but one instance.

If $\frac{3}{4}$ *of a yard of Tape cost* $\frac{1}{2}$ *of a peny, what shall one inch, that is,* $\frac{1}{36}$ *of a yard cost?*

Multiply the second by the third, the product is $\frac{1}{72}$, which divided by $\frac{3}{4}$, the quotient is $\frac{4}{216}$ of a peny, for the price of $\frac{1}{36}$ of a yard.

Otherwise,

Seeing $\frac{3}{4}$ of a yard may be turned to 27 inches: Say, if 27 cost $\frac{1}{2}$, what 1? divide $\frac{1}{2}$ by 27, it makes $\frac{1}{54}$ for the answer: which is equal to $\frac{4}{216}$, and in the least terms.

And wheresoever this may be done, to have the first and third Numbers Fractions of one denomination, the best way is to work with their Numerators, not regarding their denominators at all: As, *If* $\frac{2}{3}$ *cost* $\frac{3}{4}$, *what* $\frac{7}{3}$? *Instead* thereof write, *If* 2 *cost* $\frac{3}{4}$; *what* 7? Multiply $\frac{3}{4}$ by 7, it produceth $\frac{21}{4}$, which divided by 2, the quotient is $\frac{21}{8}$, and that is the answer in the least terms.

And all this while it should have been noted, that the Fractions are ever written in a smaller figure then the whole Numbers.

The

THE RULE OF Practice.

IN the *Golden Rule*, or *Rule of Three Direct*, I intimated; that if the first of the three Proportional Numbers given were *One*, that then the Product of the second and third numbers gives the fourth Proportional Number sought without using of any *Division*; — Also, that if the second or third of the Proportionals given were *One*, then there was no need of *Multiplication*; but dividing the greater of them by the first, the Quotient shall be the fourth Proportional sought for.

And from hence is framed this Rule of *Practice*, (by some called the Merchants Rule) which alwayes hath *One*, an ingredient in the Question, and it is no other but an *Abridgement* or *Compendium* of the *Rule of Three*, when *One* is one of the three Proportionals given.

And that such Questions that are to be resolved by this Rule may be the more readily and easily answered (Money commonly being one of the three Terms)

Terms) it is expedient that he which intendeth to make much uſe of this Rule ſhould have readily in his mind the *Even* or *Aliquot* parts of a *Pound*, of a *Shilling*, and of a *Peny*. And alſo to have in *Memory* the ſeveral *Products* of 12 (the number of *Pence* in one *Shilling*) multiplied into 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. All which are ſet down in the three ſmal *Tables* following, which ought firſt perfectly to be learned by heart, before farther Progreſs be made into this Rule.

TABLE I. *The Aliquot or Even parts of a Pound or* 20 *Shillings.*

ſh.—d.			
1—0	is the	One Twentieth $\frac{1}{20}$	of a Pound or 20 *s.*
2—0		One Tenth $\frac{1}{10}$	
2—6		One Eighth $\frac{1}{8}$	
3—4		One Sixth $\frac{1}{6}$	
4—0		One Fifth $\frac{1}{5}$	
5—0		One Fourth $\frac{1}{4}$	
6—8		One third $\frac{1}{3}$	
10—0		One half $\frac{1}{2}$	

TABLE

TABLE II. *The Aliquot or Even parts of a Shilling.*

d.	q.				
1	0	is the	$\frac{1}{12}$ One Twelfth	of a Shilling.
1	2		$\frac{1}{8}$ One Eighth	
2	0		$\frac{1}{6}$ One Sixth	
3	0		$\frac{1}{4}$ One Fourth	
4	0		$\frac{1}{3}$ One Third	
6	0		$\frac{1}{2}$ One Half.	

TABLE III. *The several Pence in a Shilling multiplied by* 12.

2	Pence multiplied by 12 produceth	24
3		36
4		48
5		60
6		72
7		84
8		96
9		108
10		120
11		132
12		144

For the working of the *Rule* of *Practice*, when the Price given is the Equal parts of a *Shilling*, this is

THE

THE RULE.

Knowing by your Table, what part of a Shilling it is, (whether $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.*) *divide the sum propounded by it, and the Quotient shall be the number of Shillings answering your Question.*

Examples

At 6 d. the Ounce, what 7625 Ounces? Six Pence is (by your Table) $\frac{1}{2}$ of a Shilling, wherefore take one half of 7625, and it is 3812 *s.* and 1 remaining, which 1 is 6 *d.* So that 7625 Ounces, will cost 3812 *s.* 6 *d.* which is reduced into Pounds, by cutting of the last figure towards the right hand of 3812, and taking the half of the other figures, which will be Pounds, and if one remain in taking of the half it is 10 *s.* ——— So the figure 2 being cut off from 3812, the half of 381 is 190 and 1 remaining, which is 190 *li.* 12 *s.* So the price of 7625 ounces will be 190 *li.* 12 *s.* 6 *d.* And so must you do for all others. As if the price be $\frac{1}{3}$ take $\frac{1}{3}$, if $\frac{1}{4}$ take $\frac{1}{4}$, as by the Examples following.

(1) At 6 *d.* the Ounce, what 7625 Ounces?

$\frac{1}{2}$	381|2 ———	6 *d.*
	190 - 12 ———	6 *d.*

(2) At 4 *d.* the yard, what 3621 yards?

$\frac{1}{3}$	120|7		
	60 *li.*	7 *s*,	0 *d.*

(3)

(3) At 3 *d*. the Gallon, what 989 Gallons?

$\frac{1}{4}$	24|7		3*d*.
	12 *li*.	7 *s*.	3 *d*.

(4(At 2 *d*. the Pound, what 1760 Pounds?

$\frac{1}{6}$	112|6		8*d*.
	56 *li*.	6 *s*.	8 *d*.

(5) At 1 *d*. 2*q*. the Ell, what 9623 Ells?

$\frac{1}{8}$	120|2 10 *d*. 2 *q*.
	60 *li*. 2 *s*. 10 *d*. 2 *q*.

(6) At 1 *d*. the Ounce what 672 Ounces?

$\frac{1}{12}$	5|6	
	2 *li*.	16 *s*.

Thus have you *Examples* when the price is *even parts* of a *Shilling*, But when they are *uneven parts* of a *Shilling*, as 5 *d*. 7 *d*. or the like, then you must do the work at two or three Operations, though in the same manner, As Pence

If the Price be	5	take for it	3 and 2
	7		4 and 3
	8		4 and 4
	9		6 and 3
	10		6 and 4
	11		6 and 3 and 2
	Pence		*d*. *d*.

Examples

Examples of these uneven Parts of a Shilling.

(1) At 5 *d.* the Gallon, what 6254 Gallons?

$\frac{1}{4}$ is 3 *d.*	1563		6 *d.*
$\frac{1}{6}$ is 2 *d.*	1042		4 *d.*
5	260l5		10 *d.*
	130 *li.*	5 *s.*	10 *d.*

(2) At 7 *d.* the Ounce, what 9271 Ounces?

$\frac{1}{3}$ is 4 *d.*	3090		4 *d.*
$\frac{1}{4}$ is 3 *d.*	2317		9 *d.*
7	540l8		1 *d.*
	270 *li.*	8 *s.*	1 *d.*

(3) At 8 *d.* the Yard, what 7952 Yards?

$\frac{1}{3}$ is 4 *d.*	2650		8 *d.*
$\frac{1}{3}$ is 4 *d.*	2650		8 *d.*
8	530l0		4 *d.*
	265 *li.*	1 *s.*	4 *d.*

(4)

(4) At 9 *d.* the Ell, what 3769 Ells?

½ is 6 *d.*	1884		6 *d.*
¼ is 3 *d.*	942		3 *d.*
9	28216		9 *d.*
	142 *li.*	6 *s.*	9 *d.*

(5) At 10 *d.* the Dozen, what 625 Dozen?

½ is 6 *d.*	312		6 *d.*
⅓ is 4 *d.*	208		4 *d.*
10	5210		10 *d.*
	26 *li.*	0 *s.*	10 *d.*

(6) At 11 *d.* the Pound, what 6952 Pound?

½ is 6 *d.*	3476		
¼ is 3 *d.*	1738		
⅙ is 2 *d.*	1158		8 *d.*
11	63712		8 *d.*
	318 *li.*	12 *s.*	8 *d.*

(7) At 12 *d.* or 1 *s.* the Ounce, what 9871 Oun.
$\frac{1}{20}$ of 20 *s.* therefore ½ 98712 is
493 *li.* 12 *s.*

If the Price of the Commodity is in Farthings, or Half pence, bring the Sum into Pence, and work as in the preceding Questions, and according to the following Examples

(1)

(1) At 1 *q.* the Pound, what 6392 Pound?

$\frac{1}{4}$	1598		
$\frac{1}{12}$	13|3		2 *d.*
	6 *li.*	13 *s.*	2 *d.*

(2) At 2 *q.* the Ell, what 3625 Ells?

$\frac{1}{2}$	1812	2 *q,*
$\frac{1}{12}$	15|1	
	7 *li.* 11 *s.* 0 *d.*	2 *q.*

(3) At 3 *q.* the Ounce, what 7321 Ounces?

$\frac{1}{2}$	3660	3 *q.*
$\frac{1}{12}$	305	
$\frac{1}{2}$	152	6 *d.*
	45|7	
	22 *li.* 17 *s.* 6 *d.* 3 *q.*	

This is the manner of working for the even parts of a Peny, but if they be uneven parts; As two pence 3 farthings, five pence 1 farthing or the like, *Work first for the even parts of a Shilling, and then for the farthings, which added the work is done.* As in these Examples.

(1)

(1) At 3 *d*. 3 *q*. the Ell, what 817 Ells?

$\frac{1}{4}$	204	3 *q*.
$\frac{1}{4}$	51	$\frac{3}{4}$
	2515	
	12 *li*. 15 *s*. 0 *d*. 3 *q*. $\frac{3}{4}$	

(2) At 4 *d*. 1 *q*. the Pound, what 7138 Pounds?

$\frac{1}{6}$	1189	8
again	1189	8
$\frac{1}{8}$	148	$8\frac{3}{4}$
	25218	$0\frac{1}{2}$
	126 *li*. 8 *s*.	0 *d*. $\frac{1}{2}$

For the even parts of a *Pound*, you must take the parts as you find them expressed in the Table as for 10 *s*. the $\frac{1}{2}$, for 5 *s*. the $\frac{1}{4}$, for 4 *s*. the $\frac{1}{5}$; as in Example

(1) At 2 *s*. 6 *d*. the Ell, what 6294 Ells?
$\frac{1}{8}$ 786 *li*. 15 *s*.

(2) At 4 *s*. the Ream, what 735 Reams?
$\frac{1}{5}$ 147 *li*.

If (in this Rule) at any time the Question consists of the parts of an Ell, *Yard*, *Pound*, *Ounce*, *Crosse*, or the like; you must deal with the whole *Ells*, *Yards*, *Ounces*, *&c.* first, and afterwards add the price of the $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$, or what other part soever it be. And thus much shall suffice for this Rule of Practice

The Rule of Fellowship.

THis Rule is useful for *Merchants*, and all such as Trade in *Companies*, with a Joynt stock; and must share a proportional part of the gains, or loss; every one according to his stock which he laid in.

The Rule is two-fold, with *equal* time; or with *unequal* time.

That which is with *equal time*, is commonly called, The Rule of Fellowship *without time*.

Of this we will first speak.

The Rule.

As the whole Joynt stock is to all the gain or losse; So is each mans particular stock, to his part of the gain, or losse.

Example 1.

Two Purchasers A. and B. buy 700 *l. a year Land for ever,* (when money is at 8, *per cent.*) *for* 14000 l. *of which A. paid* 8000 l. *and B.* 6000 l. *after* 5 *years* (money being fallen to 6 *per Cent.*) *they sell it for* 18700 l. *so there is gained* 4700 l. *how much of this must A. have?*

First for A.

Say, if 14000 gain 4700, what 8000? answer, 2685 $\frac{10000}{14000}$.

Then

Then for B.

If 14000 gain 4700, what 6000? answer $2014\frac{4000}{14000}$. As by the following operation doth appear.

(I) For A.

l. l. l.
If 14000 gain 4700, what 8000.
8

```
14000) 37600000(2685 10000/14000
           ....
         28000
          96000
          84000
          120000
          112000
            80000
            70000
            10000 remainder.
```

 (II) For

(II) For B.

l.	*l.*	*l.*
If 14000 gain	4700, what	6000
		6

14000)28200000(2014 $\frac{4000}{14000}$.

28000
20000

14000
60000

56000
4000 remainder.

Here note, That this Work might have been much abreviated, if from each of the three numbers you had cut off two Cyphers towards the right hand, as hath been formerly shewed in the Compendiums of *Multiplication* and *Division*.

Now for the proof hereof,

If you add 2685 $\frac{10000}{14000}$.
which is the sum that A. gained ;

To——2014 $\frac{4000}{14000}$.

The sum which B. gained; the sum of them is 4700.

Which is equal to the total gain.

And according to the proportion of these two Numbers : that is, as 8 to 6, or 4 to 3. So they ought to have parted the yearly Rent also, all the time they received it : that is, A. ought to have 400 *l.* yearly ; and B. 300 *l.*

Example

Example 2.

A. B. and C. joyn their moneys to make a stock of 25000 l. *of which A. laid in* 10000 l. *B.* 8000 l. *and C. put in* 7000 l. *with this (after a certain time in trading) they gained* 7500 l. *how must this be parted?*

First for A.

Say, if 25000 gain 7500, what 10000?
Or shorter, if 25 get $7\frac{1}{2}$, what 10? Multiply $7\frac{1}{2}$ by 10, it produceth 75, which divided by 25, the quotient is 3, that is, (restoring the three Cyphers) 3000 *l.* for A.

Then for B.

Say, if 25000 gain 7500, what 8000?
Or shorter, if 250 get 75, what 80?
Multiply and divide as the *Golden Rule* requires, and to the quotient restore the two Cyphers, then it will be 2400 *l.* for B.

Lastly, for C.

Say, if 250 give 75, what 70? answer 21, to which put the two Cyphers, it makes 2100 for C.
And these three 3000, 2400, and 2100, being added together, make 7500. And have that proportion as the particular stocks had: and therefore the Work is right.

(I) for A.

If 25 gain $7\frac{1}{2}$, what 10 s.

150

$7\frac{1}{2}$ $\frac{15}{2}$ $\times$ $\frac{10}{1}$

2

$\cancel{1}$
$\cancel{1}\cancel{5}\cancel{0}$ (75
$\cancel{2}\cancel{2}$

$\cancel{1}$
$\cancel{7}\cancel{5}$ (3 3000 *l.* for A.
$\cancel{2}\cancel{5}$

(II) for B.

If 24000 gain 75, what 80 ?

80

250) 6000 (24

500
1000

1000

2400 for B.

(III) for C.

If 250 gain 75, what 70?
70

250) 5250 (21

500
250

250
2100 for C.

And if instead of gaining 7500 *l.* whereby every one is supposed to have his stock, and a part of the gains; they had lost 7500 *l.* then their particular stocks had not been due to them, bnt so much as would be left after their proportional parts of the loss were abated.

Example 3.

A. B. and C. with a joynt stock of 25000 l. *gain* 7500: *of which A. gets* 3000, *B.* 2400, *C.* 2100; *what was their stock?*

This is but the Converse of the former, therefore say, if 5700 require 25000, what doth 3000 require? 10000 for A, and so work for the other two.

Many examples are of little use (except to load the Readers memory) where the Rule is so short and plain; I will therefore add no more to this part of the Rule, but immediately come to the Rule of Fellowship with time.

The Rule of Fellowſhip with time.

THis Rule is to be uſed when the times of the continuance of the particular ſtocks are unequal, and differ; ſo that here the difference of time, and alſo the difference of ſtock being both to be conſidered; it can be done no better way then by taking the *Power* of them both to be the particular ſtock; and all thoſe *Powers* added, to be the whole ſtock, that which I call the *Power* is the product of the money of every one multiplyed by his time; And then

THE RULE.

As the ſumme of thoſe Products, is to the whole gain; ſo is each particular product, to its part of the gain.

Queſtion 1.

Three Merchants A. B. C. make a ſtock of 10000 l. *of which A. layes in* 4000 *for* 3 *moneths, B.* 3000 l. *for* 6 *moneths; and C.* 3000 l. *for* 8 *moneths, with this they gain* 2000 l. *what is each mans ſhare?*

Firſt, for A. multiply 4000 by 3, it makes 12000, let that be accounted his particular ſtock.

Secondly, for B. multiply 3000 by 6, it makes 18000, his particular Stock.

Laſtly,

Laftly, for C. multiply 3000 by 8, it produceth 24000, for his ftock, add thefe, they make 54000 *l.* for the general ftock; then fay,

For A.

If 54000 give 2000, what 12000? anfwer, $666\frac{24000}{54000}$.

Then for B.

If 54000 give 2000, what 18000? anfwer, $444\frac{36000}{54000}$.

Laftly, for C.

If 54000 give 2000, what 24000? anfwer, $888\frac{48000}{54000}$.

The three Fractions may be reduced (by dividing each Numerator, and Denominator by 6000) and then the three fhares will be $444\frac{4}{9}$, $666\frac{6}{9}$, and $888\frac{8}{9}$, which altogether make 2000, as they ought.

Queftion 2.

Three Farmers, A. B. and C. lay out 1000 l. *to ftock their grounds with Cattel, of which A. put in* 200 l. *for* 6 *years*; *B. had* 300 l. *going for* 4 *years*; *and C.* 500 l. *for* 2 *years*; *at the end (by unfeafonable times) there was loft* 200 l. *which made the remain of their ftock but* 800 l. *what had each man left?*

Multiply 200 by 6, it gives 1200: Likewife, 300 by 4, it gives 1200. Laftly, 500 by 2, the product is 1000: all thefe are 3400 for the joynt ftock.

Then firft for A.

Say, if 3400 loofe 200, what 1200? anfwer, $70\frac{2000}{3400}$ for A, to which B. is equal, becaufe the power of his ftock is fo.

There-

Therefore for C.

Say, if 3400 lose 200, what 1000? answer, $58\frac{2800}{3400}$. So the 3 shares are $70\frac{20}{34}$, $70\frac{20}{34}$, and $28\frac{28}{34}$, equal to 200.

Now because A. put in 200 *l.* and lost $70\frac{20}{34}$, subtract the loss from the stock, remains $129\frac{14}{34}$.

And so doing for B, his remain will be $289\frac{14}{34}$.

And for C, his remain is $441\frac{6}{34}$. Now these three remains, $129\frac{14}{34}$, $229\frac{14}{34}$, and $441\frac{6}{34}$, make up 800 *l.* which was the whole remain.

Quest. 3.

A. rents a close for a year, to pay 80 l. *he puts into it* 200 *sheep: 2 moneths after B. puts* 40 *sheep in, and 5 moneths after that C. puts in* 100 *sheep; how much must every one pay of the rent?*

Multiply 200 by 12, it produceth	2400
And 40 by 10, produceth	400
Lastly, 100 by 5, (which is C. time) produceth	500
In all	3300

Then for A.

If 3300 pay 80, what 2400? answer, $58\frac{600}{3300}$.

Then for B.

If 3300 pay 80, what 400? answer, $9\frac{2300}{3300}$.

And for C.

If 3300 pay 80, what 500? answer, $9\frac{400}{3300}$.

The whole numbers make 79, and the broken numbers make 1. In all 80.

Note.

Note.

Whereas, hitherto we have considered onely difference of *time* and *money*; it may be noted, that there may be difference of other kinds, as *persons* or *place*; but whatsoever they are, the power of all is found like these by multiplication; and are to be wrought like these, with so many Uses of the *Golden Rule*, as the question requires. I will therefore add but one question more, which is this.

Quest. 4.

One leaves a Legacy of 900 l. *among four Kinsfolks, A. B. C. D; so as B. may have twice as much as A, and C. thrice as much as B, and D. as much and half as much as C; what is every one to have?*

Say, if A. be 1, B. is 2, C. 6, and D. 9, add these Numbers: 1, 2, 6, 9, together, they give 18, then say, If 18 require 900, what 1? Answer is 50. So A. is to have 50 *l.* B. 100 *l.* C. 300 *l.* and D. 450 *l.* which are their just parts; and altogether are equal to 900 *l.* and the work right.

Barter.

TO Barter, is to exchange one Commodity for another, the nature whereof will best appear by the resolving of some Questions.

Question 1.

Two Merchants Barter, One hath Sugar at 4 l. *the* C. *ready money, but in Barter he will have* 4 l. 13 s. 4 d. *The other hath French Wine at* 13 l. *the Hogshead ready money; at what price must he rate his Wine to equalize the others advance of his Sugar in Barter!*

Say, by the Rule of Three direct.

If 4 *l.* in Barter require 13 *s.* 4 *d.* advance, what shall 13 *l.* in Barter require ?

```
   l.      s.  d.        l.
If 4 ——— 13 — 4  what 13 ?
          12
        ———
         3 0
        1 3
      ———————
       1 6 0                        x d.
         1 3                      (x4 (4
      ———————                 x08 0 (5x0 (413
       4 8 0                   444 (xxx
     1 6 0                          x  2
     ————————                       l   s.  d.
     2 0 8 0           Answer       2 — 3 — 4
```

Question

Quest. 2.

Two Barter, one hath 3 C. ½ of Ginger at 13 d. ½ per pound. The other hath Sugar at 15 d. ¼ per pound. How much Sugar must be delivered for the 3 C. ½ of Ginger?

First, By the Rule of Three (or Practice) find what the 3 C. ½ of Ginger comes to at 13 d. ½ *per* pound, which will be found to be 22 *l.* 1 *s.* For

If 1 *l.* cost 13 *d.* ½, what 3 C. ½ cost?

Answer, 22 *l.* 1 *s.*

Secondly, Say, If 15 *d.* ½ buy 1 *l.* of Sugar, what shall 22 *l.* 1 *s.* buy?

Answer, 347 $\frac{7}{31}$.

Question 3.

Two Barter, One hath Tobacco at 14 d. per *pound, which he will Barter for Sugar at* 10 d. *per* l. *how much Tobacco must be given for* 8900 l. *of Sugar?*

First, the 8900 *l.* of Sugar at 10 *d. per* pound comes to 370 *l.* 16 *s.* 8 *d.*

Then, If 14 *d.* buy 1 *l.* of Tobacco, what number of pounds will 370 *l.* 16 *s.* 8 *d.* buy?

Answer, 6357 pound, and so many pounds of Tobacco at 14 *d.* must be given for 8900 pound of Sugar at 10 *d.*

Quest. 4.

Two Barter, One hath broad Cloth at 15 *s. the yard ready mony, for which in Barter he will have* 16 *s.* 3 *d.*

3 *d.*——*The other hath Wooll at* 2 s. 10 d. per *pound ready money? what price must his Wooll be set at in Barter to equalise the advance which he puts upon his Cloth.*

Say by the Rule of Three direct.

If 15 *s.* ready money require 1 *s.* 3 *d.* in Barter; what shall 2 *s.* 10 *d.* ready money require?

Answer, 2 *d.*——3 *q.* $\frac{1}{3}$.

So that he must rate his Wooll at 3 *s.* 3 *q.* $\frac{1}{3}$ of a farthing *per* pound.

OF
INTEREST
Simple and Compound.

IN the *Appendix* to the *Second Part* of this *Book*, I have *Tables of Compound Interest*, *Rebate* or *Discount* of *Money*, *Purchase* of *Leases* and *Annuities*, whose *Constructions* and *Uses* are there *Exemplified* by the Resolving of *Questions* sutable to each *Table*, as by having recourse thither will appear. But for that *Tables* may not alwaies be at hand, I thought it convenient here to shew how to resolve *Questions* both in *Simple* and *Compound Interest*, by which *Tables* of that nature may be *Calculated*, were there not enough already extant.

Question

Question 1.

If 100 l. *in* 12 *moneths gain* 6 l. *what shall* 625 l. *gain in* 3 *years or* 36 *moneths?*

The Proportion is,
As 100 *l.* is 6 *l.* in a year.
So is 625 *l.* to 112 *l.* 10 *s.* in a year.

Wherefore multiply 625 *l.* by 6 *l.* and divide the Product by 100, by cutting off two figures, the Quotient will be $37\frac{50}{100}$, that is, 37 *l.* 10 *s*, and this being multiplyed by 3, giveth 112 *l.* 10 *s.* as by the Work appears.

```
 l.        l.
100--------6---------625
                       6
                    ----
                   37|50
 l.    s.            3
                  ------
112---10          112|50
```

Question 2.

If 100 l. *in* 12 *moneths gain* 6 l. *what will* 236 l. 10 s. 5 d. *gain in* 16 *moneths?*

The Proportion.

As 100 *l.* is to 6 *l.* in a year,
So is 236 *l.* 10 *s.* 5 *d.* to 14 *l.* 3 *s.* 9 *d.* 3 *q.* in a year.

```
             l.   s.   d.
Multiply    236---10---5   by 6,
```

The Product is 1419---2---6,
This divide by 100, which is done by cutting off two

two figures of the Integer, leaving 14 *l.* on the left hand of the line. The figures on the right hand multiplyed by 20, and the figures (or remains) again by 12; and lastly by 4, shall in all give 14 *l.* 3 *s.* 9 *d.* 3 *q.*

Which divide by 3, and add that third part to 14 *l.* 3 *s.* 9 *d.* 3 *q.* the sum will be 18 *l.* 18 *s.* 5 *d.* 0 *q.* as by the work appeareth.

l.	*l.*	*l.*	*s.*	*d.*
100	6	236	10	5
				6

l. 14 | 19—2—6
20

s. 3 | 82
12

d. 9 {1 8} | 70 2

90
4

q. 3 | 50

l. 1 *s.*
2 4 3 2 1
l. *s.* *d.* *q.* — *l.* *s.* *d.* *q.*
14—3—9—3 — 4.14.7.1

3 3 3 3 3

	l.	*s.*	*d.*	*q.*
In a year	14	3	9	3
In $\frac{1}{3}$ of a year	4	14	7	1
In 16 moneths	18	18	5	0

By this manner of Work, *If* 417 l. 11 *s.* 8 d. *be put out at Interest for* 2 *years at* 6 l. per *Cent. it will amount unto* (*I mean the Interest*) 50 l. 2 s. 2 d. *As by the Work appears.* *li.*

li.	li.	li.	s.	d.
100	6	417	11	08
				6
		25105	10	00

	li.	s.	d.	
				120
In one year	25	01	01	1110
			2	112
In two years	50	02	02	1120
				1 4
				180

Thus these Questions are wrought by the *Single Rule of Three*; But they may be otherwise wrought by the *Golden Rule Compound of* 5 *Numbers*. Of which in that *Rule* you have an Example.

Of Compound Interest.

It may be wrought in the same manner that *Simple Interest* was, only add the increase every year as it riseth, to the Sum of the year before going, so continuing this course, till you have gone through the Numbers of years required.

Question.

What will 500 li. *amount unto, if it be forborn* 4 *years, after the rate of* 6 per Cent. *Compound Interest*.

I. First

First Work.

li.	*li.*	*li.*
100	6	500
		6
		li. 30100

First year 30 *li.*

Add this 30 *li.* found at this first work, to the 500 *li.* it makes 530 *li.* then for the

Second Work.

li.	*li.*	*li.*
100	6	530
		6
		li. 31180

Second year 31 *li.*

Which add to 530 *li.* it makes 561 *li.*

Third Work.

li.	*li.*	*li.*
100	6	561
		6
		li. 33166

Third year 33 *li.*
Which add to 561 *li.* it makes 594 *li.*

Fourth

Fourth Work.

li.	*li.*	*li.*
100	6	594
		6
		li. 35|64

For the fourth year 35 *li.*

The four *Products* of these four *Multiplications*, being added together (having respect to the figures cut off,) do make 131 *li.* which added to the Principall makes 631 *li.* $\frac{1}{10}$, and so much doth the Principal and Interest amount unto, being forborn four years, As here may be seen.

The Principal	500	
1 Product	30	00
2 Product	31	80
3 Product	33	66
4 Product	35	64
	131	10

And observing this *method*, you may resolve any *Question* for any *Number of years*, and for any *Rate of Interest*, and by this *Rule* is the *first Table* in the *Appendix* of the second part of this *Book* made. And here it will not be improper to add two of those Tables in that *Appendix* (which are there in Decimal Numbers) reduced into *Pounds*, *Shillings*, *Pence*, and *Farthings*.

The One *Shewing what any Sum of Money forborn, and number of years under* 31, *will amount or be increased unto.*

The other *shewing the present worth of any* Annuity, Rent, *or* Portion, *for any number of years to come under* 31.

The

I. TABLE.

years.	li.	s.	d.	q.
1	1	1	2	2
2	1	2	5	2
3	1	3	9	3
4	1	5	3	0
5	1	6	9	0
6	1	8	4	2
7	1	10	0	3
8	1	11	10	2
9	1	13	9	2
10	1	15	9	3
11	1	17	11	2
12	2	0	3	0
13	2	2	7	3
14	2	5	2	2
15	2	7	1	1
16	2	10	9	2
17	2	13	10	1
18	2	17	1	0
19	3	0	6	0
20	3	4	1	3
21	3	7	11	3
22	3	12	0	3
23	3	16	4	3
24	4	0	11	3
25	4	5	10	0
26	4	10	11	3
27	4	16	5	2
28	5	2	2	3
29	5	8	4	2
30	5	14	10	1

The First Table, *Shewing what any Sum of money being forborn any number of years (under 31) will be augmented unto, accounting Interest upon Interest at* 6 per Cent. per Annum.

The Use of the Table.

Question 1.

If 324 li. *be forborn for the term of* 18 *years, how much will it be increased unto? accounting* 6 li. per Cent. *Compound Interest.*

Look in the Table for 18 years and right against it you shall find 2 *li.* 17 *s.* 1 *d.* And so much will 1 *li.* or 20 *s.* be increased unto in 18 years.

Then say by the Rule of Three

If 1 *li.* give 2 *li.* 17 *s.* 1 *d.* what will 324 *li.* give?

```
        20
        --
        57 s.
        12
        --
        115
        57
        ---
        685 d.
        324
        ----
       2740
      1370
     2055
     ------
  12)221940            11 s.
     ......          (184915
                        924 li.
     12
     101
     ---
      96               li.   s.   d.
       59              924   15   00
     ----
      48
      114
     ----
      108
        60
     -----
        60
```

So that 324 *li.* being forborn 18 years will be increased unto 924 *li.* 15 *s.*

Question

Question 2.

If 156 li. 15 s. 6 d. *be forborn* 20 *years, to what will it amount.*

Against 20 years in the Table is 3 *li.* 4 *s.* 1 *d.* 3 *q.* Wherefore work by the Rule of Three as followeth.

li.	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
1	3	4	1	3	136	15	6
	20 *s.*				20		
240							
	64				2735 *s.*		
	12				12		
	129				5476		
	64				2735		
	769 *d.*				32826 *d.*		
	4				3079		
	3079 *q.*				295434		
					229782		
					984780		
					101071254		

xxxx q. xxx d.
xxxxx (1 xxx (3 xxxx(6 1
xxxxxxx154 (4xxxxx (xxxxxx(87713
xxxxxx40 xxxxxx xxxxx
xxxxx xxx 438,13

438 *li.* 13 *s.* 6 *d.* 3 *q.*

So much will 136 *li.* 15 *s.* 6 *d.* be increased to in 20 years.

II. TABLE.

years	li.	s.	d.	q.
1	0	18	10	2
2	1	16	8	0
3	2	13	5	2
4	3	9	3	2
5	4	4	3	0
6	4	14	4	1
7	5	11	7	3
8	6	4	2	1
9	6	16	0	1
10	7	7	2	1
11	7	17	8	3
12	8	7	8	0
13	8	17	0	2
14	9	5	1	3
15	9	14	3	0
16	10	2	1	1
17	10	9	6	2
18	10	16	6	2
19	11	3	2	0
20	11	9	4	3
21	11	15	3	1
22	12	0	10	0
23	12	6	0	3
24	12	11	0	0
25	12	15	8	0
26	13	0	0	3
27	13	4	2	2
28	13	8	1	2
29	13	11	9	3
30	13	15	3	2

The Second Table, *Shewing what any* Annuity Rent, *or* Pension, *being forborn any number of* years *under* 31, rebating *or* discounting *yearly, after the rate of* 6 per Cent, *Compound Interest is worth in ready Money.*

The Use of the Table.

Question 1.

What is a Lease of 25 li. per Annum *payable yearly, and to continue* 21 *years, worth in present Money?*

Look in the Table for 21 years, and against it you shall find 11 *li.* 15 *s.* 3 *d.* 1 *q.* Then say by the *Golden Rule.*

If

If 1 *li.* be worth 11 *li.* 15 *s.* 3 *d.* 1 *q.* what 25 *li.*

```
          20
        ------
         235 Shillings
          12
        ------
         473
        235
        ------
        2823 Pence.
           4
      ---------
       11292 Farthings.
          25
      -------------
       56460
      22584
      ---------
      282300

             x  x
     z       xx9x(3 d.
 z8z3øø (7ø575 (5881
  44444   xxxxx (294   1 s.   3 d.
            xxx
       294 li.     1 s.     3 d.
```

And so much is 25 *li.* a year worth in ready Money to continue 21 years.

Question

Question 2.

What is an Annuity *of* 75 *li. a year to continue* 30 *years, and to be paid yearly, worth in ready Money.*

Against 30 years in the Table you find 13 *li.* 15 *s.* 3 *d.* 2 *q.*

Then by the Rule of Three, say,

If 1 *li.* 13 *li.* 15 *s.* 3 *d.* 2 *q.*

```
        20
      ----
       275  sh.
        12
      ----
       553
      275
      ----
      3303  d.
         4
      ----
     13212  q.
        75
      ----
     66060
    92484
    ------
    990900
```

xx d.
x3xxx x54(9
9908oo (x47728 (206413
444444 xxxxxxx 1032-s.
xxxx
li. s. d.

1032——3——9——And so much is 15 *li.* a year for 30 years worth in present money.

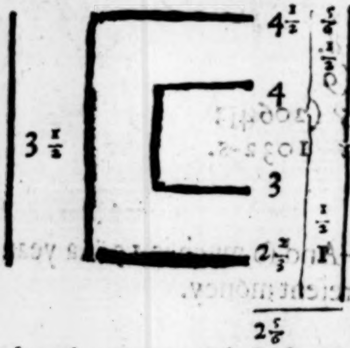
The Rule of Alligation.

THis hath its name from *binding, tying,* or *uniting* many particulars in one *Masse* or *Sum*, the nature of it will be understood in working some *Questions* or *Examples*.

Question 1.

A Corn Master would mix 4 *sorts of grain together*, viz. *Wheat at* 4 s. *the bushel, Wheat at* 4 s. 6 d. *the bushel; Rie at* 3 s. *the bushel, and Barley at* 2 s. 8 d. *the bushel; so as to make* 15 *quarters in all, to be sold at* 3 s. 6 d. *the bushel; How much must be take of each?*

Place



Place them as in the Margin, so as a greater and lesser may still be together, as $4\frac{1}{2}$, with $2\frac{2}{3}$ and 4 with 3, and place the price required by it self towards the left hand, as here you see $3\frac{1}{2}$ then in a seperated Column, note the difference between the price of a bushel of every one particular given, and a bushel of that required, as the difference betwixt $4\frac{1}{2}$ and $3\frac{1}{2}$ is 1, which must not be placed against $4\frac{1}{2}$, but against that number linked with $4\frac{1}{2}$, that is, against $2\frac{2}{3}$, and so must all the differences be ordered, as is easie to be seen in the Margine: then

THE RULE.

Multiply the whole Masse *to be made, by any particular* difference; *and divide the product by the sum of all the* differences, *the quotient shall be the just quantity of that* particular kind, *whose price standeth against the difference you wrought with.*

Example.

First turn the quarters into bushels, by saying, 8 times 15 is 120, then for the quantity of the first sort at $4\frac{1}{2}$: multiply 120 by $\frac{5}{6}$, the product is 100, which divided by $2\frac{5}{6}$, the quotient is $35\frac{5}{17}$ bushels of that sort at 4 *s.* 6 *d.* and working so for every of the

the other; they will be found to be thus.

At 4 s. 6 d.	$35\frac{5}{17}$
At 4 s.	$21\frac{3}{17}$
At 3 s. 6 d.	$21\frac{3}{17}$ bushels.
At 2 s. 8 d.	$42\frac{6}{17}$
In all	120

4.6
2-8
1.10

Now to prove this right; First, multiply the whole Masse 120 bushels, by the desired price $3\frac{1}{2}$ s. omitting the denominations, the sum is 420 s.

Then secondly, multiply $38\frac{5}{17}$ by $4\frac{1}{2}$, it is, $158\frac{14}{17}$

And $21\frac{3}{17}$ by 4, it produces	$84\frac{12}{17}$
And $21\frac{3}{17}$ by 3, it makes	$63\frac{9}{17}$
And $42\frac{6}{17}$ by $2\frac{2}{3}$ is	$112\frac{16}{17}$
In all	420

And so much all should be worth in shillings. And therefore the question is rightly resolved.

Question 2.

One hath 6 sorts of Fruits at several prices, Dates at 2 s. *Almonds at* 1 s. 4 d. *Currants at* 10 d. *Raisins at* 5 d. *Prunes at* 4 d. *and Figs at* 3 d. *the pound; and would take of every sort some to make a mixed quantity of* 30 l. *weight to sell one with another for* 9 d. *the pound, how much must he take of each?*

Having

	24	6	
	16	5	
	10	4	
9			30
	5	1	
	4	7	
	3	15	
	Sum	38	

Having placed the Numbers and their differences, and the sum of those differences distinctly as hath been shewed before, and may be seen by the Figure in the Margine; the Work is evermore like that in the former question. So 38 is the first number in the *Golden Rule*; 30, the second (which that it may not be forgotten, may be set at the right side of the figure) and every particular difference, as 6, 5, 4, &c. is the third in the Rule, to be repeated till all the differences have been employed.

So 30, multiplyed by 6, produceth 180, which divided by 38, the quotient is $4\frac{22}{38}$ of a pound weight: and so much must be taken of Dates, at 24 *d.*

Secondly, 5 times 30 is 150, which divided by 38, the quotient is $3\frac{16}{38}$ for Almonds. And working after the same manner with 4, 1, 7, 15, their respective quantities will be found to be these;

	pounds	38 parts.
Dates	4,	28
Almonds	3,	36
Currants	3,	6
Raisins	0,	30
Prunes	5,	20
Figgs	11,	32
In all	26,	152

That

That is 26 $\frac{152}{38}$, and the reduction of the fraction will make it 30, as it ought to be, and by comparing the prices of these particulars added, with the price of 30 *li.* weight, at 9 *d. per l.* weight, which makes 470 *d.* this may be proved like the former.

But that the Reader may be perfect in it, I will do it here also, as followeth.

Say first, 24 times 4 is 96, and 24 times 28 is 672; for the first, set them thus : 96, 672

And 16 times 3 is 48 and 16 times 36 is 576	48, 576
And 10 times 3 is 30, and 10 times 6 is 60	30, 60
And 5 times 30 is 150	00, 150
And 4 times 5 is 20, and 4 times 20 is 80	20, 80
Lastly, 3 times 11 is 33, and 3 times 33 is 96	33, 96
In all	227, 1634

Now this 1634 being the sum of the Numerators of Fractions, whose common Denominator is 38, must be divided by 38, and the quotient will be 43, which added to the whole number 227 : the sum is 270. And so much is 30, multiplyed by 9, which shews the work to be right.

The Combination, or linking of Numbers may be varied at pleasure, as whereas above I linked 24 and 3, also 16 with 4, and 10 with 3, it might have been 24 with 5, and 16 with 4, and 10 with 3. Or 24 with 4, and 16 with 3, and 10 with 4, of which diversity of linking would follow diversity

of

of solutions, but all true, as the Reader may easily prove by himself.

Likewise, if the Numbers to be linked were 3, 5, 7, or any odd Number, one of them may be linked to two severally, to make the work even.

Example 3.

If the Numbers were 12, 10, 8, 6, and 4, and the mean or common price required were 9, you might first link them as you see here, taking 12 twice, or else you might take any other twice as you shall like; and so the work will be every way right, though not the same; if the differences be rightly set off, and orderly used, as is taught before in the first question.

	12	5
	12	3
	10	1
9		
	8	1
	6	3
	4	4

Question 3.

A Goldsmith would mix 3 *sorts of Silver, A. B. C, A. is* 10 d. *weight better; B.* 7 d. *weight better; and C.* 4 d. *weight better, to make an Ingot of* 50 *li. weight, which should be in fineness* 8 d. *weight better: How much must be taken of each?*

Set

Set them, their differences, and the summe of their differences as in the Margin.

	10	4	
	10	1	
8			50
	7	2	
	4	2	
	9		

1. Then first 50 multiplyed by 4 is	200
and divided by 9, the quotient is	9
2. 50 multiplyed by 2,	50
and divided by 9, the quotient is	9
3. 50 multiplyed by 2,	100
and divided by 9, the quotient is	9
4. And again, the same	100
	9
In all	450
	9

Which is equal to 50, the quantity required,

Now the first fraction multiplyed by 10, (omitting the Denominator) is 2000

The second also by 10 gives	500
The third by 7 makes	700
The last by 4 makes	400
In all	3600

That is, $\frac{3600}{9}$, which is equal to 400, and if the whole Ingot 50, be multiplyed by the betternesse required, namely, by 8, they shall produce 400 also: So this is proved.

In every Alligation, or linking of two numbers, this is evident, that if the summe of the numbers linked be greater then the mean number required taken so many times as there are numbers to be linked, the question would be absurd; and the resolution thereof impossible. And this shall serve for the Rule of *Alligation*.

The Rule of False Position.

THis Rule serves to resolve Questions, which are not presently fit for the *Golden Rule*; and therefore in stead of the true Number which is sought: Suppose any number great or small, and make trial of it, whether it resolve the Question without any error; if so, it is the true Number: if not,

not, note what error is at the end of the work, and whether that error be *too much*, or *too little*; if *too much*, mark it thus +, but if *too little*, thus —

Then suppose again another Number, (it imports not whether it be nearer or further off) and try as before, and mark that error also with + or —, according as you find it to be, either *more* or *less*.

And then this is

THE RULE.

Multiply the first Position *by the second* Error, *and the second* Position *by the first* Error, *and (*if the errors be both + or both —) Substract *the* lesser *product from the* greater, *and keep the remain for a* Dividend, *and the difference of the errors for the* Divisor; *the quotient of that division is the true Number required.*

But if the errors be one +, the other —, then the summe of the products added together must be the Dividend: *and the summe of the* errors, *the* Divisor; the rest of the work is the same as before.

Question 1.

A man is to drive 48 *young Turkies* 40 *miles: and for every Turkie which comes alive to the end of the Journey, he is to receive* 3 d. *but for every one which dyes by the way, he is to pay* 6 d. *At the end he received* 72 d. *How many dyed by the way?*

Let the first supposition be
That by the way there dyed 20
For them he was to pay 120 *d.* and for 28 which lived, he was to receive 84 *d.*
so he paid more then he received 36 *d.*
and should have got clear 72 *d.*

Add 108

Wherefore the first error is —108

Let the second supposition be 10
For these he paid 60 *d.* and for the rest he received 114 *d.* the difference is 54, and should be 72: so the second error is —18

Now 20 multiplyed by 18, produceth 360
and 10 by 108 produceth 1080
and the difference is 720 for the *Dividend*, likewise the difference of the errors is 90 for the *Divisor*, and the quotient is 8, which is the true Number of those he lost by the way. As may be proved by trial.

Question 3.

If it were required to make up a pound Sterling *of shillings and groats only: and so as the number of* groats *may be to the Number of* shillings, as 7 to 1; *How many shillings must there be?*

First, suppose the shillings 4
then the groats must be equal to 16 *s.*
that is 48 groats; but the shillings
taken 7 times are 28, to which
48 should be equal, but is + 20

Secondly,

Secondly, suppose the shillings 2
then the groats (making 18 s.) are 25,
which should be equal to 7 times 2, but is + 40
Multiply 4 by 40, product is 160, then
Multiply 2 by 20, the product is 40, which taken from 160, rests for the Dividend 120
And the difference of errors is 20

Lastly, 120 divided by 20, the quotient is 6
The number of shillings therefore is 6
And the number of groats is 42
But as 7 to 1, so is 6 times 7 which is 42, to 6 times 1, which is 6 : so the work is done.

Question 3.

If there be 4 several weights, A.B.C.D, of which D. is 24 ounces, and C. is double to B, and triple to A, and D. with twice A. is double to C, and quadruple to B: How much doth every one of these weights weigh.

First, suppose A. to be 8
then D with twice A is 24, and 16, that is 40, of which C. being the half is 20, and B 10.
Now thrice A. is 24, to which C. should be equal, but is −4

Secondly, let A be supposed A 4
then D. more, twice A, is 32, and C. 16
and B. is 8, but thrice A. is 12, to which 16 should be equal, but is +4

 Then

Then 8 multiplied by 4, gives 32, and 4 by 4, produceth 16: both these products give 48 for the Dividend; and the summe of the errors (because the first is —, the other +) gives 8, for the Divisor; and the quotient will be 6, to which A. is equal, and twice A. more D. is 36, of which C. being half is 18, and B. is 9, and thrice A. is equal to C, namely, 18, and all right.

Whereas the first error is equal here to the second, it follows that the Positions were equally false; and therefore their difference which is 4, being parted into two equal parts, 2 and 2, if 2 be taken from 8, the remain is the true number 6, or if 2 be added to 4, (which was the second position) the summe will be also 6.

And further, whensoever the errors be one +, the other —, though they be not equal; yet then if the difference between the positions be parted into two parts, which are in proportion one to another, as the two errors are one to another respectively: then if the first part be taken from the first position (if that be the greater) or add to it (if it be the less) the same number required is thereby had.

As, let the last question be resumed,	
And let the first position for A. be	15
Then the first error will be	—18
Then let the second position be	3
And so the second error will be	+6
And the difference of position is	12

which divided into two parts 9 and 3, which have that proportion one to another as have the errors 18 and 6, then if the first part 9, be taken from the first position 15, there remains the true Number 6. Or else

elſe if the ſecond part 3, be added to the ſecond poſition 3: thereby alſo is made the true Number 6.

The way of parting 12 (or any other) into two parts proportional with the errors is eaſily done by the *Golden Rule*, thus:

As the ſumme of the Errors 24,
is to the difference of poſitions 12;
So is the greater error 18,
to the greater part required, namely, 9.

Many other queſtions are in other Books exemplified and wrought by this Rule; but ſeeing I intend not to write a great Book; and alſo becauſe ſome of thoſe queſtions may be eaſily reſolved without this Rule, I will add no more: onely mention one of thoſe queſtions.

If there be a Ciſtern with 4 Cocks, which holds 8 Barrels of water, and the firſt cock will run it all out in 6 hours, the ſecond in 4, the third in 3, and the laſt in 2 hours: in what time ſhall all of them run it out?

If the firſt in 6 hours run	8
the ſecond in the ſame time would run	12
the third	16
the laſt	24
In all	60

Then ſay, If 60 require 6, what 8?

The anſwer $\frac{48}{60}$, that is $\frac{4}{5}$ of an hour; in which time all the 4 Cocks together would run out all the 8 Barrels of water.

The Rule of Ceres and Virginum.

THis is the most uncertain, and unnecessary Rule in Arithmetick: being seldome used except in sporting questions to puzzle young Beginners, with easie problems; such as follow.

Question 1.

A Caterer bought 8 *birds of two sorts, as Geese and Hens for* 20 s. *the Geese cost* 4 s. *a piece, the Hens* 2 s. *a piece; How many did he buy of each sort?*

This may be done by the Rule of False; and also thus: multiply the whole number 8, into the least price 2, it produceth 16, which taken from the whole price 20, rests 4 for a *Dividend*; which divided by 2, which is the difference of the particular prices, the quotient is 2, for the number of *Geese*; and 6 must be for the *Hens*: the proof is easie.

Question 2.

If 21 *Persons,* Men, Women, *and* Children *spend* 26 *shillings; so that every* man *payes* 2 s. *every* woman 1 s. *every* child 6 d. *How many is there of each sort.*

THE

THE RULE.

Multiply the number of Persons *by the* least expence, *and take the product of it from the* whole expence, *the rest shall be the* Dividend; *which divided by the* difference *betwixt the* greatest *and* least *particular* expences: *the quotient is a* Number, *which the Number of men (or they which spend most) comes near to; but cannot exceed: or if the said* Dividend *be divided by the summe of the* greatest *and* least *expences, the quotient is a Number, then which the Number of men (or those which spend most) cannot be much lesse.*

So here 21 multiplyed by *6 d.* that is, by $\frac{1}{2}$, the product is 10 $\frac{1}{2}$, which taken from 26, rests 15 $\frac{1}{2}$, for the Dividend: and then taking $\frac{1}{2}$ from 2, rests 1 $\frac{1}{2}$ for the Divisor, and the quotient is $\frac{31}{3}$, which is something more then 10, the Numbers of men therefore must be but 9.

Then turn the Dividend, and the Divisor both into whole Numbers, by multiplying them by the common Denominator 2, so they reduced will be 31 and 3, as before is to be seen in the quotient.

Multiply the Divisor 3, by 9, (which is the Number of *Men*) the product is 27, which taken from 31, (which is the reduced Dividend) the remain is 4, for the Number of *Women*; and the *Children* must be 8.

Example

Example 1.

9 men at 2 *s.* each	18 *s.*
4 women at 1 *s.* each	4
8 children at 6 *d.* each	4
In all 21	In all 26

But the number of men may be also 8, which multiplyed by the reduced Divisor 3; product is 24, which taken from 31, the remain is 7 for the women; and then the children must be 6.

Example 2.

8 men at 2 *s.* each	16
7 women at 1 *s.* each	7
6 children at 6 *d.* each	3
In all 21	In all 26

Or the number of men may be 7, which multiplyed by 3, produceth 21, which taken from 31, remains 10 for the women, and 4 children.

Example 3.

7 men at 2 *s.* each	14 *s.*
10 women at 1 *s.* each	10
4 children at 6 *d.* each	2
In all 21	In all 26

So

So is already seen 3 various solutions of this question, which makes this Rule the less to be regarded. But further, the number of men may be 10, and not more, for if you put them 11, that multiplyed by 3, produceth 33, which is greater then 31, from which it should be taken, but I say it may be 10, and then there is only one woman, and five children; this confirmes the former part of the Rule.

Now for the later part, if the Dividend 31 be divided by the summe of the two extream expences (reduced by doubling as the Dividend is) 4, the quotient will be $7\frac{3}{4}$. And the men may be 7, as hath been shewed; but they may be also but 6, and fewer they cannot be: as 6 men, 13 women, and 2 children; for if you put them 5, that multiplyed by 3, produceth 15, which taken from 31, there remains 16 for the women, and so there should be no children: which is contrary to the supposition.

And further, because the quotient was $7\frac{3}{4}$, the number of men might be so, if pure Arithmetical division be only regarded: and then the women also are in number $7\frac{3}{4}$, and the children $5\frac{1}{2}$, as may easily be tryed; I need not exemplifie it.

Question 3.

If there be an Exhibition of 900 l. per annum *to* 30 *persons: some Clerks, some Messengers, and some Door-keepers, at* 60 l. *each Clerk,* 40 l. *each Messenger, and* 20 l. *each Door-keeper; How many must there be of each sort?*

Multiply (according to the Rule) 30 by 20, the product is 600, which taken from 900, remains

300

300 for the Dividend; and 60 want 20, that is 40, for the Divisor: and the quotient is 7 $\frac{1}{2}$, and more the Clerks cannot be; Also divide by 60 more 20, that is 80, quotient is 3 $\frac{3}{4}$, and much fewer the Clerks cannot be.

Not to stand upon the Fractions (in this case of dividing men) the Clerks may be 7, 6, 5, 4 or 3: and the Messengers 1, 3, 5, 7 or 9, and the Door-keepers 22, 21, 20, 19 or 18, that the Clerks cannot (in whole Numbers) be more then 7, or lesse then 3, (may thus be proved; First, let them be 8, then 8 times 40 is 320, which is more then 300, out of which it should be taken: Secondly, let them be 2, then 2 times 40 is 80, out of 300 remains 220, which divided by 20, gives the quotient 11, for the Messengers; so the Clerks and Messengers being 13, the remain thereof to 30, namely, 17, must be Door-keepers.

but,

2 Clerks at 60 *l.* each	120 *l.*
11 Messengers at 40 *l.* each	480
17 Door-Keepers at 20 *l.* each	340
In all 30	In all 940

Which is 40 *l.* too much, therefore the Clerks cannot be two.

Note.

It may be asked, why the remain 220 should be divided by 20: whereas the like remain in the former Example, namely, 163 was taken (without any division) absolutely for the number of Women, or

middle

middle number? I answer, although the greatest or first Number being found, (as here to be 2) the residue of 2 to 30, might be rightly parted into two fit parts in the same manner as the first question of this Rule was resolved, or else by the Rule of False: yet to give further satisfaction, the cause of this is, the difference betwixt the two lesser expences, was there $\frac{1}{2}$, which (before the division was reduced to 1, which neither multiplies nor divides any Number, but leaves it the same: whereas, in this last, the middle expence (or exhibition) being 40, and the least 20, the difference of them was 20, by which dividing the Remain of the last substraction: the quotient is ever the Number of the middle persons. Which may serve as an addition to the Rule, where the sorts of things are but three.

Question 4.

If there be 10 *persons of four several Countries,* English, French, Dutch *and* Spanish, *to pay a Debt of* 1000 l. *So that every English man payes* 50 l. *every French man* 70 l. *every Dutch man* 130 l. *and every Spaniard* 150 l. *How many is there of each?*

The Dividend (according to the former Rule) is 500.

Now to make the Divisor, take his summe that payes least (namely 50) out of each of the other three 150, 130 and 70, and the Remains will be 100, 80, and 20.

Add the first and last for the Divisor, it is 120.

And the quotient will be $4\frac{2}{12}$, and the Spaniards cannot be more. Secondly,

Secondly, add the first and second together for the Divisor, it is 180, and the quotient is $2\frac{14}{18}$, and the Spaniards cannot be less.

I mean, they cannot be much more than 4, or less then 2: and therefore, seeing any one solution will serve, let them be 3, and by that multiply 100, and take the product out of 500, there remains 200 for a second Dividend, which divided by (the second remain) 80, the quotient is $2\frac{1}{2}$: therefore Dutchmen are 2, which multiplyed by 80, make 160, take that out of 200, there remains 40 for a third Dividend: which divided by (the third remain) 20, the quotient is 2 for the Frenchmen also; and consequently the English must be 3, because all of them are 10: But the Spaniards may be also 4 or 2.

Example.

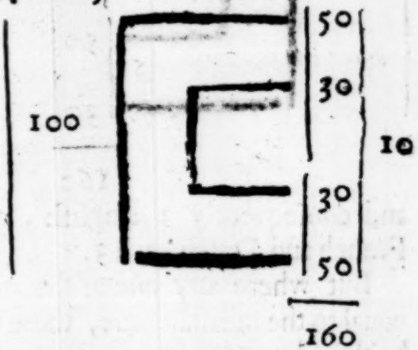
4	Spaniards at 150 *l.* each	600
1	Dutchman at 130 *l.*	130
1	Frenchman at 70 *l.*	70
4	English at 50 *l.* each	200
10	In all	1000

2	Spaniards at 150 *l.* each	300
3	Dutch at 130 *l.* each	390
3	French, at 70 *l.* each	210
2	English at 50 *l.* each	100
10	In all	1000

The

The reaſon why the Spaniards and Engliſh, as alſo the Dutch and French are equal in number, is becauſe their payments differ equally from 100, which is the mean ſum with which 10 men ſhould pay 1000 *l.* and making it ſo, this queſtion, and many other of this nature may be anſwered by the Rule of *Alligation*: thus,

If 160 give 10, what 50? Anſwer is $3\frac{20}{160}$, (that is in this caſe 3) for the Spaniards, and as many for the Engliſh, becauſe their reſpective differences from



100, the one 50 more; the Engliſh 50 leſs, are equal.

And alſo, becauſe the other two differences 30 and 30 are equal: the Number of the French is equal to the Number of the Dutch.

But both thoſe numbers together are 4, becauſe 3 Spaniards, and 3 Engliſh, taken out of 10, the remain muſt be 4.

Wherefore the Number of the French is 2, and the Dutch alſo are 2.

Or thus,

Accounting the men in the ſame order as before.

If 160 require 10, what 30?

Anſwer is $1\frac{160}{160}$, (that is, in this caſe 2) for the Spaniards; and conſequently 2 Engliſh, and therefore the French and Dutch each 3.

But where any one of the particular ſummes is equal to the mean ſumme, there this cannot ſo well be done by *Alligation.*

Example.

If one ſhould buy 12 *Loaves of bread for* 12 pence, *ſo that ſome might be* two peny; *ſome* peny, *ſome* half-peny; *and ſome* farthing *loaves: and it be required to know how many he muſt buy of each.*

Then becauſe of 12 *loaves* for 12 *pence*, the mean price is 1, but one of the particulars being alſo 1, there ſhould be no peny loaves, becauſe there is no difference betwixt the mean *price* and a *peny.*

But

But it may be found by the Rule of *Ceres* and *Virginum*, to be either.

4 two peny loaves	8 pence
2 peny loaves	2 pence
2 half-peny loaves	1 peny
4 farthing loaves	1 peny
In all 12 loaves.	In all 12 pence.

Or else,

3 two peny loaves	*6 d.*
4 peny loaves	4
3 half-peny loaves	$1\frac{1}{2}$
2 farthing loaves	$0\frac{1}{2}$
In all 12	In all 12

N Extraction

Extraction of Roots.

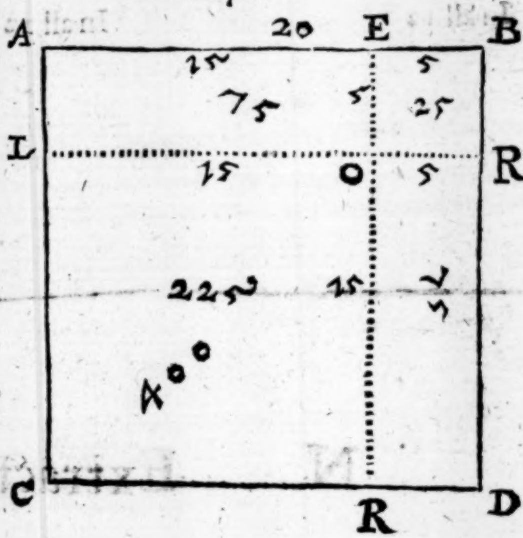
And first for the *SQUARE ROOT*, that is, a square Number being given, to find the root or side of it in a Number: which root or side, being multiplied into it self, must therefore produce that square Number.

The doing of this is shewed in (almost) all books of Arithmetick; and the reason of it (in some) which is taken from the fourth proposition of the second book of *Euclide*; which saith,

If a right line be divided by chance; the squares made of the parts, together with the Rectangle made of the parts twice, is equal to the square of the whole.

Example

Let the line A B, be divided by chance in the point E, it is manifest that the square of AB, that is to say the



square

square A B D C, is equal to the square of L O, that is, of A E, and to the square of E B, and to the two rectangle figures A O and D O, that is, the rectangle A O (which is made of the parts A E, and B E) twice according to the Proposition.

Now let A B, be supposed 20, A E 15, and B E 5.

Then the square of A B, (which is made by multiplying the root 20 into it self) is equal to 400.

And the square A E, that is, 15 times 15 is equal to	225
And the square of B E is 5 times 5	25
And the rectangle A O, 5 times 15	75
And the rectangle D O, 5 times 15	75
In all	400

which is equal to the square of A B, as before.

If therefore a square Number, as 334084 be given to have the root extracted : first, make a point over 4, (the place of Unity) and another over 0, the second figure from it; and so another over 3, the second figure from 0, towards the left hand, observing the like order still, if there were more secondary figures, point them all as in the Margine;

$$\begin{array}{l} 3\dot{3}4\dot{0}8\dot{4} \\ 25 \\ \\ \\ \\ 084084 \\ 749 \end{array}$$

Then look at the figures under the first point towards the left hand which are 33, and take the greatest square root of them, which is 5, (for *6* times *6* is 36) and say 5 times 5 is 25, which take out of 33, there remains . . . .

Then multiply the Root 5 by 20, it

gives 100 for a Divisor; and the Dividend is 840, namely, the figures which reach to the next point, and so the quotient might be 8, but must be but 7, because the square of the quotient, being now 49, must (together with 7 times the Divisor) be taken out of 840.

And the remainder will be 009184

Add this last quotient 7, being put to the former quotient 5, after the manner used in plain Division, it will be 57, which multiply still by 20, the product is 1140 for a new Divisor, and the Dividend (because there remains but one point) is all the figures 9184.

9184

0000

And the quotient can be but 8, and must be so much; for 8 times 1140 is 9120

and 8 times 8 is 64

In all 9184

Which taken out of the remain 9184, there now remains nothing: which shews the square 334084 is justly resolved, and puting the last quotient 8 to the two former, 5 and 7, the whole will be 578, which is the true *Root* required.

And the parts of it, if they be 3, are 500, 70, and 8. Or if but two 570 and 8. And the truth may be either way proved by adding the squares and twice the rectangles of the parts; for the sum of them

them shall be equal to the whole square 334084, as hath been shewed before.

It is ever certain that there shall be as many figures, or places of figures in the root, as there are points in the square, ordered as before.

And by reading this little seriously, may any one be able to find the root of any other square whatsoever: the first operation in all being to take the greatest square contained under that point next the left hand, and put the Root thereof for the first figure in a quotient.

The Secondary operation, must be repeated so often as there are remaining points; as hath been plainly shewed in the fore-going Example.

I will therefore add no more, but give you two other squares, and their roots, leaving the Reader to extract them himself.

So if there were given the square 8836
The root of it according to the former practice may be found to be 94

Or let there be given the square 1522756
The root of that will be 1234.

Here followeth a Table of Roots and their Squares from 1, to 1000.

1 Cent.		1 Cent.		1 Cent.	
R.	Square	R.	Square	R.	Square
1	1	31	961	61	3721
2	4	32	1024	62	3844
3	9	33	1089	63	3969
4	16	34	1156	64	4096
5	25	35	1225	65	4225
6	36	36	1296	66	4356
7	49	37	1369	67	4489
8	64	38	1444	68	4624
9	81	39	1521	69	4761
10	100	40	1600	70	4900
11	121	41	1681	71	5041
12	144	42	1764	72	5184
13	169	43	1849	73	5329
14	196	44	1736	74	5476
15	225	45	2025	75	5625
16	256	46	2116	76	5776
17	289	47	2209	77	5929
18	324	48	2304	78	6084
19	361	49	2401	79	6241
20	400	50	2500	80	6400
21	441	51	2601	81	6561
22	484	52	2704	82	6724
23	529	53	2809	83	6889
24	576	54	2916	84	7056
25	625	55	3025	85	7225
26	676	56	3136	86	7396
27	729	57	3249	87	7569
28	784	58	3364	88	7744
29	841	59	3481	89	7921
30	900	60	3600	90	8100

1 Cent.		1 Cent.		1 Cent.	
R.	Square	R.	Square	R.	Square
91	8281	121	14641	151	22801
92	8464	122	14884	152	23104
93	8649	123	15129	153	23409
94	8836	124	15376	154	23716
95	9025	125	15625	155	24025
96	9216	126	15876	156	24336
97	9409	127	16129	157	24649
98	9604	128	16384	158	24964
99	9801	129	16641	159	25281
100	10000	130	16900	160	25600
101	10201	131	17161	161	25921
102	10404	132	17424	162	26244
103	10609	133	17689	163	26569
104	10816	134	17956	164	26896
105	11025	135	18225	165	27225
106	11236	136	18496	166	27556
107	11449	137	18769	167	27889
108	11664	138	19044	168	28224
109	11881	139	19321	169	28561
110	12100	140	19600	170	28900
111	12321	141	19881	171	29241
112	12544	142	20164	172	29584
113	12769	143	20449	173	29929
114	12996	144	20736	174	30276
115	13225	145	21025	175	30625
116	13456	146	21316	176	30976
117	13689	147	21609	177	31329
118	13924	148	21904	178	31684
119	14161	149	22201	179	32041
120	14400	150	22500	180	32400

2 Cent.		2 Cent.		2 Cent.	
R.	Square	R.	Square	R.	Square
181	32761	211	44521	241	58981
182	33124	212	44944	242	58564
183	33489	213	45369	243	59049
184	33856	214	45796	244	59536
185	34225	215	46225	245	60025
186	34596	216	46656	246	60516
187	34969	217	47089	247	61009
188	35344	218	47524	348	61504
189	35721	219	47961	249	62001
190	36100	220	48400	250	62500
191	36481	221	48841	251	63001
192	36864	222	49284	252	63504
193	37249	223	49729	253	64009
194	37636	224	50176	254	64516
195	38025	225	50625	255	65025
196	38416	226	51076	256	65536
197	38809	227	51529	257	66049
198	39204	228	51984	258	66564
199	39601	229	52441	259	67081
200	40000	230	52900	260	67600
201	40401	231	53361	261	68121
202	40804	232	53824	262	68644
203	41209	233	54289	263	69169
204	41616	234	54756	264	69696
205	42025	235	55225	265	70225
206	42436	236	55696	266	70756
207	42849	237	56169	267	71289
208	43264	238	56644	268	71824
209	43681	239	57121	269	72361
210	44100	240	58600	270	72900

2 Cent.		3 Cent.		3 Cent.	
R.	Square	R.	Square	R.	Square
271	73441	301	90601	331	109561
272	73984	302	91204	332	110224
273	74529	303	91809	333	110889
274	75076	304	92416	334	111556
275	75625	305	93025	335	112225
276	76176	306	93636	336	112896
277	76729	307	94249	337	113569
278	77284	308	94864	338	114244
279	77841	309	95481	339	114921
280	78400	310	96100	340	115600
281	78961	311	96721	341	116281
282	79524	312	97344	342	116964
283	80089	313	97969	343	117649
284	80656	314	98596	344	118336
285	81225	315	99225	345	119025
286	81796	316	99856	346	119716
287	82369	317	100489	347	120409
288	82944	318	101124	348	121104
289	83521	319	101761	349	121801
290	84100	320	102400	350	122500
291	84681	321	103041	351	123201
292	85264	322	103684	352	123904
293	85849	323	104329	353	124609
294	86436	324	104976	354	125316
295	87025	325	105625	355	126025
296	87616	326	106276	356	126736
297	88309	327	106929	357	127449
298	88804	328	107584	358	128164
279	89401	329	108241	359	128881
300	90000	330	108900	360	129600

3 Cent.		4 Cent.		4 Cent.	
R.	Square	R.	Square	R.	Square
361	130321	391	152881	421	177241
362	131044	392	153664	422	178084
363	131769	393	154449	423	178929
364	132496	394	155236	424	179776
365	133225	395	156025	425	180625
366	133956	396	156816	426	181476
367	134689	397	157609	427	182329
368	135424	398	158404	428	183184
369	136161	399	159201	429	184041
370	136900	400	160000	430	184900
371	137641	401	160801	431	185761
372	138384	402	161604	432	186624
373	139129	403	162409	433	187489
374	139876	404	163216	434	188356
375	140625	405	164025	435	189225
376	141376	406	164836	436	190096
377	142129	407	165649	437	190969
378	142884	408	166464	438	191844
379	143641	409	167281	439	192721
380	144400	410	168100	440	193600
381	145161	411	168921	441	194481
382	145924	412	169744	442	195364
383	146689	413	170569	443	196249
384	147456	414	171396	444	197136
385	148225	415	172225	445	198025
386	148996	416	173056	446	198916
387	149769	417	173889	447	199809
388	150544	418	174724	448	200704
389	151321	419	175561	449	201601
390	152100	420	176400	450	202500

4 Cent.		5 Cent.		6 Cent.	
R.	Square	R.	Square	R.	Square
451	203401	481	231361	511	261121
452	204304	482	232324	512	262144
453	205209	483	233289	513	263169
454	206116	484	234256	514	264196
455	207025	485	235225	515	265225
456	207936	486	236196	516	266256
457	208849	487	237169	517	267289
458	209764	488	238144	518	268324
459	210681	489	239121	519	266361
460	211600	490	240100	520	270400
461	212521	491	241081	521	271441
462	213444	492	242064	522	272484
463	214369	493	243049	523	273529
464	215296	494	244036	524	274576
465	216225	495	245025	525	275625
466	217156	496	246016	526	276676
467	218089	497	247009	527	277729
468	219024	498	248004	528	278784
469	219961	499	249001	529	279841
470	220900	500	250000	530	280900
471	221841	501	251001	531	281961
472	222784	502	252004	532	283024
473	223729	503	253009	533	284089
474	224676	504	254016	534	285156
475	225625	505	255025	535	286225
476	226576	506	256036	536	287296
477	227529	507	257049	537	288369
478	228484	508	258064	538	289444
479	229441	509	259081	539	290521
480	230400	510	260100	540	291600

5 Cent.		5 Cent.		6 Cent.	
R.	Square	R.	Square	R.	Square
541	292681	571	326041	601	361201
542	293764	572	327184	602	362404
543	294849	573	328329	603	363609
544	295936	574	329476	604	364816
545	297025	575	330625	605	366025
546	298116	576	331776	606	367236
547	299209	577	332929	607	368449
548	300304	578	334084	608	369664
549	301401	579	335241	609	370881
550	302500	580	336400	610	372100
551	303601	581	337561	611	373321
552	304704	582	338724	612	374544
553	305809	583	339889	613	375769
554	306916	584	341056	614	376996
555	308025	585	342225	615	378225
556	309136	586	343396	616	379456
557	310249	587	344569	617	380689
558	311364	588	345744	618	381924
559	312481	589	346921	619	383161
560	313600	590	348100	620	384400
561	314721	591	349281	621	385641
562	315844	592	350464	622	386884
563	316969	593	351649	623	388129
564	318096	594	352836	624	389376
565	319225	595	354025	625	390625
566	320356	596	355216	626	391876
567	321489	597	356409	627	393129
568	322624	598	357604	628	394384
569	323761	599	358801	629	395641
570	324900	600	360000	630	396900

6 Cent.		6 Cent.		7 Cent.	
R.	Square	R.	Square	R.	Square
631	398161	661	436921	691	477481
632	399424	662	438244	692	478864
633	400684	663	439569	693	480249
634	401959	664	440896	694	481636
635	403226	665	442225	695	483025
636	404495	666	434556	696	484416
637	405769	657	444889	697	485809
638	407044	668	446224	698	487204
639	408321	669	447561	699	488601
640	409600	670	448900	700	490000
641	410881	671	450241	701	491401
642	412164	672	451584	702	492804
643	413449	673	452929	703	494209
644	414736	674	454276	704	495616
645	416025	675	455625	705	497025
646	417316	676	456976	706	498436
647	418609	677	458329	707	499849
648	419904	678	459684	708	501264
649	421201	679	461041	709	502681
650	422500	680	462400	710	504100
651	423801	681	463761	711	505521
652	425104	682	465124	712	506944
653	426409	683	466489	713	508369
654	427716	684	467856	714	509796
655	429025	685	469225	715	511225
656	430336	686	470596	716	512656
657	431649	687	471969	717	514089
658	432964	688	473344	718	515524
659	434281	689	474721	719	516961
660	435600	690	476100	720	518400

7 *Cent.*		7 *Cent.*		8 *Cent.*	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
721	519841	751	564001	781	609961
722	521284	752	565504	782	611524
723	522729	753	567009	783	613089
724	524176	754	568516	784	614656
725	525625	755	570025	785	616225
726	527076	756	571536	786	617796
727	528529	757	573049	787	619369
728	529984	758	574564	788	620944
729	531441	759	576081	789	622521
730	532900	760	577600	790	624100
731	534361	761	579121	791	625681
732	535824	762	580644	792	627264
733	537289	763	582169	793	628849
734	538756	764	583696	794	630436
735	540225	765	585225	795	632025
736	541696	766	586756	796	633616
737	543169	767	588289	797	635209
738	544644	768	589824	798	636004
739	546121	769	591361	799	638401
740	547600	770	592900	800	640000
741	549081	771	594441	801	641601
742	550564	772	595984	802	643204
743	552049	773	597529	803	644809
744	553536	774	599076	804	646416
745	555025	775	600625	805	648025
746	556516	776	602176	806	649636
747	558009	777	603729	807	651249
748	559504	778	605284	808	652864
749	561001	779	606841	809	654481
750	562500	780	608400	810	656100

8 *Cent.*		8 *Cent.*		8 *Cent.*	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
811	657721	841	707281	871	758641
812	659344	842	708964	872	760384
813	660969	843	710649	873	762129
814	662596	844	712336	874	763876
815	664225	845	714025	875	765625
816	665856	846	715716	876	767376
817	667489	947	717409	877	769129
818	669124	848	719104	878	770884
819	670761	849	720801	879	772641
820	672400	850	722500	880	774400
821	674041	851	724201	881	776161
822	675684	852	725904	882	777924
823	977329	853	727609	883	779689
824	678976	854	729316	884	781456
825	680625	855	731025	885	783225
826	682276	856	732736	886	784996
827	683929	857	734449	887	786769
828	685584	858	736164	888	788544
829	687241	859	737881	889	790321
830	688900	860	739600	890	792100
831	690561	861	741321	891	793881
832	692224	862	743044	892	795664
833	603889	863	744769	893	797449
834	695556	864	746496	894	799236
835	697225	865	748225	895	801025
836	698896	866	749956	796	802816
837	700569	867	751689	897	804609
838	702244	868	753424	898	806404
839	703921	869	755161	899	808201
840	705600	870	756900	900	810000

9 Cent.		9 Cent.		9 Cent.	
R.	Square	R.	Square	R.	Square
901	811801	931	866761	961	923521
902	813604	932	868624	962	925444
903	815409	633	870489	963	927369
904	817216	934	872356	964	929296
905	819025	935	874225	965	931225
906	820836	936	876096	966	933156
907	822649	937	877969	967	935089
908	824464	938	879844	968	937024
909	826281	939	881721	969	938961
910	828100	940	883600	970	940900
911	829921	941	885481	971	942841
912	831744	942	887364	972	944784
913	833569	943	889249	973	946729
914	835396	944	891136	974	948676
915	837225	945	893025	975	950625
916	839056	946	894916	976	652576
917	840889	947	896809	977	954529
918	842724	948	898704	978	956484
919	844561	949	900601	979	958441
920	846400	950	902500	980	960400
921	848241	951	904401	981	962361
922	850084	952	906304	982	964324
923	852929	953	908209	983	966289
924	853776	954	910116	984	968256
925	855625	955	912025	985	970225
926	857476	956	913936	986	972196
927	859329	957	915849	987	974169
928	861184	958	917764	988	976144
929	863041	959	919681	999	978121
930	864900	960	921600	990	980100

9 Cent.		9 Cent.	
R.	Square	R.	Square
991	982081	996	992016
992	984064	997	994009
993	986049	998	996004
994	988036	999	998001
995	990025	1000	1000000

Extraction of the Cube Root.

A *Cube* is a ſolid, or Body contained within ſix equal ſquares; and may be fitly repreſented by a Die.

When a Cube is given, as is 435519512; point the Number as you ſee in the place of Unity, and every third figure after; then ſee what is the root of the greateſt Cube contained under the firſt point toward the left hand; that is, in 435, it will be found 7, (for 8 times 8, taken 8 times is 512, which is too much) put this 7 for the firſt figure in the quotient, having taken the Cube thereof, which is 343,

out of 435: thus; 435519512
343

And the remain will be 092519512

O And

And so the first work is done.

For the second, take the square of the quotient, that is *49*, which multiply by 300, the product is 14700
to which add 30 times 7, that is ::210

which makes the Divisor 14910
And the Dividend is 92519

So the quotient might be *6*, but must be but 5, because the Cube of the new quotient, and 210 times the square of the said quotient must be allowed in this work, as followeth.

The remain is 92519512
multiply the first product 14700 by the second quotient 5, and they produce 73500 — 78875
then multiply 210 by the square of 5, it makes :5250
to which add the cube of 5 ..::125

It makes in all 78875

Which taken from 92519 there remains 13644512
and the quotient is 75, whose square 6625, multiplyed by 300
the product is 1687500
secondly, 30 times 75 is ::2250

And the new Divisor is 1710000

And

And the third quotient figure is 8, by which multiply 1687500, it is	1350000
Likewise, 2250, multiplyed by the square of 8, (64) is	144000
And the Cube of 8 is	::: 512
Altogether are	13644512

Which taken from the second remain, there remains now thirdly nothing. And therefore the last quotient 8, put to the former two, it is 758 for the whole Root, as may be tryed by multiplying 758 into it self; and the product again by 758, then the last product shall be equal to the whole Cube, which was given at first to be resolved.

This one Example is sufficient for the general understanding of the manner of working, wherefore I will add no more Examples, but (as I did in the square Root, so in this, I will) give you two or three numbers with their Roots, leaving the practice of them to the Learner. Thus,

$$\text{Of}\left\{\begin{array}{l}389017\\56430125\\961504803\\12895213625\end{array}\right\}\left\{\begin{array}{r}73\\405\\987\\2345\end{array}\right\}\text{is the Root.}$$

Here followeth a Table of Cube Roots, from 1 to 1000.

1 *Cent.*		1 *Cent.*		1 *Cent.*	
R.	*Cube*	*R.*	*Cube.*	*R.*	*Cube*
1	1	31	29791	61	216981
2	8	32	32768	62	238328
3	27	33	35937	63	293047
4	64	34	39304	64	262244
5	125	35	42825	65	274625
6	216	36	48656	66	287496
7	343	37	50653	67	300753
8	512	38	54872	68	314432
9	729	39	55419	69	329199
10	1000	40	64000	70	333000
11	1331	41	68921	71	357911
12	1728	42	74088	72	373348
13	2197	43	79507	73	389017
14	2744	44	85184	74	405224
15	3375	45	91125	75	411875
16	4096	46	97336	76	438976
17	4913	47	103823	77	456533
18	5832	48	110592	78	474522
19	6859	49	117649	79	493039
20	8000	50	125000	80	512000
21	9261	51	135651	81	531441
22	10648	52	140608	82	550408
23	12167	53	148877	83	571787
24	13824	54	157464	84	592604
25	15625	55	167375	85	614125
26	17576	56	175616	86	636056
27	19683	57	185193	87	648303
28	21972	58	195112	88	681472
29	24389	59	205379	89	705669
30	27000	60	216000	90	729000

1 Cent.		1 Cent.		1 Cent.	
R.	Cube	R.	Cube	R.	Cube
91	753571	121	1771561	151	3442951
92	778688	122	1815848	152	3511808
93	804357	123	1860867	153	3581577
94	830584	124	1906624	154	3652264
95	857375	125	1953125	155	3723875
96	884736	126	2000376	156	3796416
97	915673	127	2048383	157	3869893
98	941192	128	2097152	158	3944312
99	970299	129	2146689	159	4019679
100	1000000	130	2197000	160	4096000
101	1030301	131	2248091	161	4173281
102	1061208	132	2299968	162	4251528
103	1092727	133	2352637	163	4230744
104	1124856	134	2406104	164	4410944
105	1157625	135	2460375	165	4492125
106	1191016	136	2515856	166	4574296
107	1225043	137	2570353	167	4657463
108	1259712	138	2628072	168	4741632
109	1295029	139	2685619	169	4826809
110	1331000	140	2744000	170	4913000
111	1367631	141	2803221	171	5000211
112	1404928	142	2864288	172	5088448
113	1442897	143	2924207	173	5177717
114	1481544	144	2985984	174	5268024
115	1520875	145	3027525	175	5359375
116	1560896	146	3112136	176	5451776
117	1601613	147	3176523	177	5545233
118	1643032	148	3241792	178	5639752
119	1685159	149	3307949	179	5735339
120	1728000	150	3375000	180	5832000

2 Cent.		2 Cent.		2 Cent.	
R.	Cube	R.	Cube.	R.	Cube
181	5929741	211	9393931	241	13997521
182	6028568	212	9528128	242	14172448
183	6128487	213	9663597	243	14348907
184	6229504	214	9800344	244	14526684
185	6331625	215	9938375	245	14705125
186	6434856	216	10077696	246	14886936
187	6539203	217	10218313	247	15069223
188	6644672	218	10360232	248	15252992
189	6751269	219	10503459	249	15438249
190	6859000	220	10648000	250	15625000
191	6967871	221	10793861	251	15813251
192	7077888	222	10941048	252	16003008
193	7189057	223	11089567	253	16194277
194	7301384	224	11239424	254	16387064
195	7414875	225	11390625	255	16581375
196	7529536	226	11543176	256	16777216
197	7645373	227	11697083	257	16974593
198	7762392	228	11852452	258	17173512
199	7882599	229	12008989	259	17473979
200	8000000	230	12167000	260	17576000
201	8120601	231	12326391	261	17779581
202	8242408	232	12487168	262	17984728
203	8365427	233	12649337	263	18191447
204	8489664	234	12812904	264	18399744
205	8615125	235	12977875	265	18609625
206	8741816	236	13144256	266	18821096
207	8869743	237	13312053	267	19034063
208	8999912	238	13481272	268	19248832
209	9129329	239	13651919	269	19465109
210	9261000	240	13824000	270	19683000

2 Cent.		3 Cent.		3 Cent.	
R.	Cube	R.	Cube	R.	Cube
271	19902511	301	27270901	331	36264691
272	20123648	302	27543608	332	36594368
273	20346417	303	27818127	333	36926037
274	20571024	304	28094464	334	37259704
275	20796875	305	28372625	335	37595375
276	21024576	306	28652616	336	37933076
277	21253933	307	28934443	337	38272753
278	21484952	308	29218112	338	38614472
279	21717639	309	29503629	339	38958219
280	21952000	310	29791000	340	39304000
281	22188041	311	30080231	341	39651821
282	22425768	312	30371328	342	40001688
283	22665187	313	30664297	343	40353607
284	22906304	314	30959144	344	40707584
285	23149125	315	31255875	345	41063625
286	23393656	316	31554496	346	41421736
287	23539903	317	31855013	347	41781923
288	23897872	318	32157432	348	42144192
289	24137569	319	32461759	349	42508549
290	24389000	320	32768000	350	42875000
291	24642171	321	33076161	351	43243551
292	24897088	322	33386248	352	43614208
293	25143757	323	33698267	353	43986977
294	25412184	324	34012224	354	44361864
295	25672363	325	34328125	355	44738875
296	25934336	326	34645976	356	45118016
297	26198073	327	34965783	357	45499293
298	26353592	328	35287552	358	45882712
279	26730899	329	35611289	359	46268279
300	37000000	330	35937000	360	46656000

3 Cent.		4 Cent.		4 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
361	47045881	391	59776471	421	74618461
362	47439928	392	60236288	422	75151448
363	47832147	393	60698457	423	75686967
364	48228544	394	61162984	424	76225024
365	48627125	395	51629875	425	76765625
366	49027896	396	62099136	426	77308776
367	49430863	397	62570773	427	77854483
368	49836032	398	63044792	428	78402752
369	50243409	399	63521199	429	78953589
370	50653000	400	64000000	430	79507000
371	51064811	401	64481201	431	80062991
372	51478848	402	64964808	432	80621568
373	51895117	403	65450827	433	81182737
374	52313624	404	65939264	434	81746504
375	52734375	405	66430125	435	82312875
376	53157376	406	66923416	436	82881856
377	53582633	407	67419143	437	83453453
378	54010152	408	67917312	438	84027672
379	54439939	409	68417929	439	84604519
380	54872000	410	68921000	440	85184000
381	55306341	411	69426531	441	85766121
382	55742968	412	69934528	442	86350888
383	56181887	413	70444997	443	86938307
384	56623104	414	70957944	444	87528384
385	57066625	415	71473375	445	88121125
386	57512456	416	71991296	446	88716536
387	57960603	417	72511713	447	89314623
388	58411072	418	73034632	448	89915392
389	58863869	419	73560059	449	90518849
390	59319000	420	74088000	450	91125000

4 Cent.		5 Cent.		5 Cent.	
R.	Cube	R.	Cube	R.	Cube
451	91733851	481	111284641	511	133432831
452	92345408	482	111980168	512	134217728
453	92959677	483	112678587	513	135005697
454	93576664	484	113379904	514	135796744
455	94196375	485	114084125	515	136590875
456	94818816	486	114791256	516	137388096
457	95443993	487	115501303	517	138188413
458	96071912	488	116214272	518	138991832
459	96702579	489	116930169	519	139798359
460	97336000	490	117649000	520	140608000
461	97972181	491	118370771	521	141420761
462	98611128	492	119095488	522	142246648
463	99252847	493	119823157	523	143055667
464	99897344	494	120553784	524	143877824
465	100544625	495	121287375	525	144703125
466	101194696	496	122023936	526	145531576
467	101847563	497	122763473	527	146363183
468	102503232	498	123505992	528	147197952
469	103161709	499	124251499	529	148035889
470	103823000	500	125000000	530	148877000
471	104487111	501	125751501	531	149721291
472	105154048	502	126506008	532	150568768
473	105823817	503	127263527	533	151419437
474	106496424	504	128024064	534	152273304
475	107171875	505	128787625	535	153130375
476	107050176	506	129554216	536	153990656
477	108531333	507	130323843	537	154854153
478	109215352	508	131096512	538	155720872
479	109902239	509	131872229	539	156590819
580	110592000	510	131651000	540	157464000

5 Cent.		5 Cent.		6 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
541	158340421	571	186169411	601	217081801
542	159220088	572	187149248	602	218167208
543	160103007	573	188132517	603	219256227
544	160989184	574	189119224	604	220348864
545	161878625	575	190109375	605	221445125
546	162771336	576	191102976	606	222545016
547	163667323	577	192100033	607	223648543
548	164566592	578	193100552	608	224755712
549	165569149	577	194104539	609	225866529
550	166375000	580	195112000	610	226981000
551	167284151	581	166122941	611	228099131
552	168196608	582	197137368	612	229220928
553	169112377	583	198155287	613	230346397
554	170031464	584	199176704	614	231475544
555	170953875	585	200202625	615	232608375
556	171879616	586	201230056	616	233744896
557	172808683	587	202262003	617	234885113
558	173741112	588	203297472	618	236029032
559	174676879	589	204336469	619	237176659
560	175616000	590	205379000	620	238328000
561	176558481	591	206425071	621	239483061
562	177504328	592	207474688	622	240641848
563	178453547	593	208527857	623	241804367
564	179306144	594	209584584	624	242970624
565	188262125	595	210644875	625	244140625
566	181221496	596	211708736	626	245314376
567	182154263	597	212776173	627	246491883
568	183150432	598	213847192	628	247673152
569	184220009	599	214921799	629	248858189
570	185193000	600	216000000	630	250047000

6. Cent.		6 Cent.		7 Cent.	
R.	Cube	R.	Cube	R.	Cube
631	251239591	661	288804781	691	329939371
632	252435968	662	290117528	692	331373888
633	253636137	663	291434247	693	332812557
634	254840104	664	292754944	694	334255384
635	256047875	665	294079625	695	335702375
636	257259456	666	295408296	696	337153536
637	258474853	667	296740963	697	338608873
638	259694072	668	298077632	698	340068392
639	260917119	669	299418309	699	341532099
640	262144000	670	300763000	700	343000000
641	263374721	671	302111711	701	344472101
642	264609288	672	303464448	702	345948408
643	265847707	673	304821217	703	347428927
644	267089984	674	306182024	704	348913664
645	268336125	675	307546875	705	350402625
646	269586136	676	308915776	706	351895816
647	270840023	677	310288733	707	353393243
648	272097792	678	311665752	708	354894912
649	273359449	679	313046839	709	356400829
650	274625000	680	314432000	710	357911000
651	275894451	681	315821241	711	359425431
652	277167808	682	317214568	712	360944128
653	278445077	683	318611987	713	362467097
654	279726264	684	320813504	714	363994344
655	281011375	685	321419125	715	365525875
656	282300416	686	322828856	716	367061696
657	283593393	687	324242703	717	368601813
658	284890312	688	325660672	718	370146232
659	286191179	689	327082769	719	371694959
660	287496000	690	328509000	720	373248000

9
9
81
9
729

7 Cent.		7 Cent.		8 Cent.	
R.	*Cube*	*R.*	*Cube*	*R.*	*Cube*
721	374805361	751	423564751	781	476379541
722	376367048	752	425259008	782	478211768
723	377933067	753	426957777	783	480048687
724	379503424	754	428661064	784	481890304
725	381078125	755	430368875	785	483736625
726	382657176	756	432081216	786	485587656
727	384240583	757	433798093	787	487443403
728	385828352	758	435519512	788	489303872
729	387420489	759	437245479	789	491169069
730	389017000	760	438976000	790	493039000
731	390617891	761	440711081	791	494913671
732	392223168	762	442450728	792	496793088
733	393832837	763	444194947	793	498677257
734	395446904	764	445943744	794	500566184
735	397065375	765	447697125	795	502459875
736	398688256	766	449455096	796	504358336
737	400315553	767	451217663	797	506261573
738	401947272	768	452984832	798	508169592
739	403583419	769	454756609	799	510082399
740	405224000	770	456533000	800	512000000
741	406869021	771	458314011	801	513922401
742	408518488	772	460099648	802	515849608
743	410172407	773	461889917	803	517781627
744	411830784	774	463684824	804	519718464
745	413493625	775	465484375	805	521660125
746	415160936	776	467288576	806	523606616
747	416832723	777	469097433	807	525557943
748	418508992	778	470910952	808	527514112
749	420189749	779	472729139	809	529475129
750	421875000	780	474552000	810	531441000

8. Cent.		8 Cent.		7 Cent.	
R.	Cube	R.	Cube	R.	Cube
811	533411731	841	594823321	871	660776311
812	535387328	842	596947688	872	663054848
813	537367797	843	599077107	873	665338617
814	539353144	844	601211584	874	667627624
815	541343375	845	603351125	875	669921875
816	543338496	846	605495736	876	672221376
817	545338513	847	607645423	877	674526133
818	547343432	848	609800192	878	676836152
819	549353259	849	611960049	879	679151435
820	551368000	850	614125000	880	981472000
821	553387661	851	616295051	881	683797841
822	555412248	852	618470208	882	686128968
823	557441767	853	620650477	883	688465387
824	559476224	854	622835864	884	690807104
825	561515625	855	625026375	885	693154125
826	563559976	856	527222016	886	695506456
827	565609283	857	629422793	887	697864103
828	567663552	858	631628712	888	700227072
829	569722789	859	633839779	889	702595369
830	571787000	860	636056000	890	704969000
831	573856191	861	638277381	891	707347971
832	575930368	862	640503928	892	709732288
833	578009537	863	642735647	893	712121957
834	580093704	864	644972544	894	714516984
835	582182875	865	647214625	895	716917375
836	584277056	866	649316896	896	719323136
837	586376253	867	651714363	897	721734273
838	588480472	868	653972032	898	724150792
839	590589719	869	656234909	899	726572699
840	592704000	870	658503000	900	729000000

9 Cent.		9 Cent.		9 Cent.	
R.	Cube	R.	Cube	R.	Cube
901	731432701	931	806954491	961	887503681
902	733870808	932	809557569	962	890277218
903	736314327	933	812166237	963	893056347
904	738763264	934	814780504	964	895841344
905	741217625	935	817400375	965	898632125
906	743677416	936	820025856	966	901428696
907	746142643	937	822656953	967	904231063
908	748613312	938	825293672	968	907039232
909	751089429	939	827936019	969	909853209
910	753571000	940	830584000	970	912673000
911	756058031	941	833237621	971	915498611
912	758550528	942	835896888	972	918330048
913	761048497	943	838561807	973	921167317
914	763551944	944	841232384	974	924010424
915	766060875	945	843908625	975	926859375
916	768575296	946	846590536	976	929714176
917	771095213	947	849278123	977	932574833
918	773620632	948	851971392	978	935441352
919	776151559	949	854670349	979	938313739
920	778688000	950	857375000	980	941192000
921	781229961	951	860085351	981	944076141
922	783777448	952	862801408	982	946966168
923	786330467	953	865523177	983	949862087
924	788889024	954	868250664	984	952763904
925	791453125	955	870983875	985	955671625
926	794022776	956	873722816	986	958585256
927	796597983	957	876467493	987	961504803
928	799178752	958	879217912	988	964430272
929	801765089	959	881974079	989	967361669
930	804357000	960	884736000	990	970299000

9 Cent.		9 Cent.	
R.	Cube	R.	Cube
991	973242271	996	988047936
992	976191488	997	991026973
993	979146657	998	994011992
994	982107784	999	997002999
995	985074875	1000	1000000000

The

Some uses of the Square and Cube Root.

Uses of the Square Root.

WHat the *Square* and *Cube Roots* are, and how to *extract* them, hath already been taught, and for more ease and expedition, there are *Tables* ready calculated, both of the *Square* and *Cube Roots*, from 1 to 1000, we come now to shew some uses thereof, which in some measure will appear in the *Propositions* following.

PROPOSITION. I.

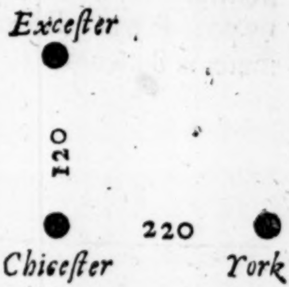
Admit the height of the Wall of a Fort or Castle to be scaled, be 30 *foot, and the bredth of the Trench about the Fort be* 40 *foot, I demand of what length a Scaling-Ladder should be, justly to reach from the edge or brow of the Trench, to the top of the Wall.*

By the 47th: of the first Book of *Euclids Elements*, it is demonstrated; that, *the square of the Hypotenusal of all right angled plain Triangles is equal to the squares of the 2 other sides*; I therefore to resolve this Proposition, square the height of the *Wall* which is 30, *facit* 900; also I square the bredth of the *Trench* which is 40 *facit* 1600, these two

two added together make 2500, the square root whereof is 50, and so long must a Scaling Ladder be made to reach from the edge of the *Trench* to the top of the *Wall*.

PROPOSITION. II.

There be two Towns, as Chichester *and* York, *which lie North and South one from another, and their distance is* 220 *miles, and* Excester *lieth directly West from* Chichester 120 *miles; I desire to know the distance of* York *from* Excester.



Square 120, the distance of *Excester* and *Chicester*, it maketh 14400, likewise square 220, the distance of *York* and *Chichester*, *facit*, 48400, these two numbers added together make 62800, whose square root extracted (or found in the Table) will be $250\frac{1}{2}$ near, and so many miles is *Excester* distant from *York*.

Use of the Cube Root.

ONe chief use of the Cube Root, is to find out a proportion between, like Solids, such are Spheres, Cubes, and such like; as in the Proposition following.

PROPOSITION.

If a Bullet of Brasse of 4 inches Diameter, weigh 9 pound, what shall a Bullet of Brass weigh, whose Diameter is 8 inches?

Cube 4 the Diameter of the lesser Bullet, makes 64, likewise Cube the Diameter of the greater Bullet 8, makes 4608. This done, say by the Rule of proportion; If the Cube number 64, give 9 *li.* weight, what shall the Cube number 4608 give? Multiply and divide, you shall find 72, and so many pounds will a Bullet of Brass weigh, whose Diameter is 8 inches.

The End of the First Part.

8
8
64
8
512

DECIMAL ARITHMETICK.

The Second Part.

Containing

The Grounds and Reason

thereof; The Use of this kind of Artificial (or Decimal) way of working, illustrated by divers Examples, in all the most usual Rules of Arithmetick.

By *Will. Leybourn.*

LONDON,

Printed Anno Dom. 1668.

DECIMAL ARITHMETICK.

The Second Part.

HAving in the firſt Part of this Book exemplified the Art of *Vulgar Arithmetick* both in *whole Numbers* and *Fractions*; We come now to treat of DECIMAL ARITHMETICK, which teacheth how to perform (as it were) in *Whole Numbers*, all that the former did effect by *Fractions*. I will not inſiſt upon the *antiquity* or *excellency* of this kind of *Arithmetick*, but come immediately to the *practice* thereof, and ſhall therefore premiſe theſe Propoſitions following.

Propoſition 1.

A vulgar Fraction being given, how to reduce the ſame into a Decimal.

The Rule.

To the Numerator *of the Fraction given, add what*

 number

number of Cyphers *you please ; then divide the* Numerator *by the* Denominator, *the quotient shall be the* Decimal Fraction *required.*

Example 1.

Let it be required to reduce $\frac{4}{17}$ *into a Decimal* : first, to the Numerator 4, add five Cyphers, so will it be 400000, divide this number by the Denominator 17, and the quotient will be .23529, which is the Decimal required.

And here note, that *Decimal Fractions* are not written in a smaller figure with a line between them, as *vulgar Fractions* are, but of the same figure, only there must be a Comma or point put between the whole Number and the Fraction, and that is the distinction.

Example 2. *If you would expresse* 235 $\frac{4}{17}$ *in a decimal way,* it must be written as followeth.

By the last example you find that $\frac{4}{17}$ reduced to a decimal, was .23529, therefore 235 $\frac{4}{17}$ must be written thus : 235,23529. In decimal fractions the Numerator is only expressed, and the Denominator onely intimated ; for this RULE is general, *Of how many figures soever the* Numerator *of a decimal fraction doth consist, of so many* Cyphers *with a* Unite *before them, doth the* Denominator *of the same fraction consist.* So this Decimal 12,625, if it were written in a vulgar way, would be 12.$\frac{625}{1000}$, but in a decimal, only 12,625, the Comma or point between 12 and 625 distinguisheth the *whole number*

ber from the *fraction*, and the fraction 625 consisting of three figures, intimates that the Denominator thereof must consist of three Cyphers and an unite before them ; So the decimal before expressed, 235, 23529, if it were written in the vulgar way, would be $235\frac{23529}{100000}$.

But it sufficeth to expresse in Decimals the Numerators only, and omit the Denominators, the Denominators of all Decimal fractions being either 10, 100, 1000, 10000, 100000, &c. according to the number of figures contained in the Numerators.

According to this Rule, you shall find that

$\frac{4}{5}$	will be in Deci-	.80000
$12\frac{3}{7}$	mals by adding	12.42857
$132\frac{9}{17}$	5 Cyphers.	132.52941

And by this means, all manner of Fractions of *Coyns*, *Weights* and *Measures*, may be reduced from vulgar Fractions, to decimal Fractions ; as by the next Proposition will appear.

Proposition 2.

How to express English-Coin *in Decimal Numbers*.

Let it be required to express 9 shillings (which is $\frac{9}{20}$ of a pound *Sterling*) in a decimal ; To the Numerator 9 add two Cyphers, making it 900, which divide by 20, the quotient is, .45, for the decimal of 9 *s*. So the decimal of 13 *s*. will be 65, and so for any number of shillings.

¶ Here note, that in the Reduction of Vulgar fractions into Decimals, that many times the first, second or third places of the Decimal fractions

are Cyphers, as in the following Table the Decimal of one farthing is .00104167, and the reason is, because if you reduce $\frac{1}{960}$ into a Decimal (for one farthing is the 960 part of a pound *Sterling)* you shall by adding of Cyphers to the Numerator find the Quotient to be 104167, but two Cyphers must be placed before it; because dividing 1000000 by 960, the place of Unites in the Divisor at the first demand extendeth unto the third Cypher in the Dividend, for in reducing of Vulgar fractions to Decimals, this is,

A general Rule.

That if the place of Unites *in the* Divisor, *at the first demand extend but unto the first of the* Cyphers *annexed to the* Numerator *of the fraction, there must be no* Cypher *put before in the Quotient, but if the place of* Unites *extend unto the second* Cypher *added, then one* Cypher *must be placed before in the Quotient, if unto the third* Cypher, *then two Cyphers must be placed before in the Quotient, &c.*

According to which Rule, if you make trial you shall find that the Decimal of 7 *s.* will be .35, the Decimal of 5 *d.* will be .02083333, the Decimal of two farthings will be .00208333, as in the Table.

By these Rules last delivered are the ensuing Tables of *English Money, Weight* and *Measure* composed, and the like may be done for a forreign *Coin*, &c. according as every mans occasions shall require.

The

The TABLE of *English* *Coin* in Decimals.

English Coin.	
Sh. 19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

D. 11	.04583333
10	.04166667
9	.0375
8	.03333333
7	.02916667
6	.025
5	.02083333
4	.01666667
3	.0125
2	.0833333
1	.04166667
F. 3	.003125
2	.00208333
1	.00104167

Troy Weight in Decimals.

O. 11	.91666667
10	.83333333
9	.75

8	.66666667		
7	.58333333		
6	.5		
5	.41666667		
4	33333333	*Gr.* 23	.00399395
3	25	22	.00381944
2	16666667	21	00364583
1	08333333	20	00347222
		19	00329861
P. 19	07916667	18	003125
18	075	17	.00295139
17	07083333	16	00277778
16	06666667	15	00260417
15	0625	14	00243056
14	05833333	13	00225694
13	05416667	12	00208333
12	05	11	00190972
11	04583333	10	.00173611
10	04166667	9	0015625
9	0375	8	00138889
8	03333333	7	00121528
7	02916667	6	00104166
6	025	5	00086805
5	02083333	4	.00069444
4	01666667	3	.00052083
3	0125	2	00044722
2	00833333	1	00017311
1	00416667		

Avoir-

Avoirdupois great weight in Decimals.

3 *qu.*	.75	7	0625
2 *qu.*	5	6	.05357143
1 *qu.*	25	5	04464286
27	24107142	4	03571428
26	23214285	3	02678571
25	.22321428	2	01785714
24	21428571	1	.00892857
23	20535714	*Oun.* 15	.00837053
22	19642857	14	0078125
21	1875	13	00725446
20	.17857143	12	00669643
19	16964286	11	00613839
18	16071428	10	.00558035
17	15178571	9	00502232
16	14285714	8	00446429
15	13392857	7	00390625
14	125	6	00334821
13	11607143	5	00279018
12	.10714286	4	00223214
11	09821428	3	.00167411
10	08928571	2	00111607
9	08035714	1	00055804
8	07141857	3 *qu.*	00041853
		half	00027902
		1 *qu.*	00013951

Avoir-

Avoirdupois little weight in Decimals.

Ounces	15	.937
	14	875
	13	8125
	12	75
	11	6875
	10	.625
	9	5625
	8	5
	7	4375
	6	375
	5	3125
	4	25
	3	1875
	2	125
	1	.0625
Draws.	15	.05859375
	14	0546875
	13	05078115
	12	046875
	11	0 4296875
	10	.0390625
	9	.03515625
	8	03125
	7	02734375
	6	0234375
	5	01953125
	4	015625
	3	01171875
	2	0078125
	1	.00390625
3 qu.		00292969
half		00195312
1 qu.		00097656

Liquid Measures in Decimals.

P.	7	.875
	6	75
	5	625
	4	5
	3	375
	2	25
	1	125
3 qu.		09375
haf.		0625
1 qu.		.03125

Dry

Dry Measures in Decimals.

Bushels.	7	.875
	6	75
	5	625
	4	5
	3	.375
	2	25
	.1	.125

Pecks.	.3	.09375
	2	0625
	1	03225
3 *qu.*		.0234375
half		015625
1 *qu.*		.0078125

Pints.	3	0058594
	2	0039063
	.1	.0019531

Long Measures, the Integers being yards and ells in Decimals.

qu.	3	.75
	2	5
	1	.25
Naile	3	.1875
	2	125
	1	.0625

3 *qu.*	046875
half.	03125
1 *qu.*	.015625

Time in Decimals.

Mo.	11	.916667
	10	833333
	9	75
	8	666667
	7	833333
	6	5
	5	416667
	4	333333
	3	25
	2	166667
	1	.083333

Da.	30	082193
	29	097454
	28	076714
	27	073973
	26	071233
	25	068495

24	.065755
23	063016
22	060274
21	057536
20	054795
19	052055
18	049316
17	046577
16	043837
15	041097
14	038357
13	035617
12	032877
11	030137
10	027397
9	024657
8	.021918
7	029178
6	016438
5	013698
4	010959
3	0082192
2	0054795
1	.0027397

Dozens in Decimals.

De.	11	.9166667
	10	8333333
	9	75
	8	6666667
	7	5833333
	6	.5
	5	4166667
	4	3333333
	3	25
	2	1666667
	1	.0833333
Pa.	11	.076388
	10	0694444
	9	0625
	8	0555555
	7	0486111
	6	.0416667
	5	0347222
	4	0277778
	3	0208333
	2	0138889
	1	.0069444

The

The use of the fore-going Tables.

THe *Tables* preceding are in number nine; The first being of *English Coyn*; The second of *Troy weight*; The third of *Avoirdupois great weight*. The fourth of *Avoirdupois little weight*. The fifth of *Liquid Measures*. The sixth of *Dry Measures*. The seventh of *Long Measures*. The eighth of *Time*, and the ninth of *Dozens*: These several Tables are made by the Rules immediately going before them, and their use is to expresse in *Decimal numbers* either *Money*, *Weight*, or *Measure*, as by the following *Propositions* will appear.

PROP. I.

How by the Table *to express* English Coin *in Decimals.*

The first of the nine Tables is for this purpose; therefore if you would expresse either *shillings*, *pence* or *farthings* in Decimal numbers, you must repair to the first Table, which is of *English Coin*, and there against 13 shillings you shall finde .65, which is the Decimal of 13 shillings, also against seven pence

pence you shall find .02916667, which is the Decimal representing 7 pence: Also against 2 farthings you shall find .00208333, which is the Decimal answering to 2 farthings, and the like is to be done for any other number of shillings, pence, or farthings.

But if it be required to find the Decimal of divers Denominations of Coin in one sum, as of shillings, pence, and farthings together, you must add the Decimals of all the particulars together, and the sum of them shall be the Decimal sought.

Examples

If you would know the Decimal of 13 *s.* 7 *d.* 2 *q.* in one number, you must first look in the Table for the Decimal of 13 *s.* which is .65, and set that down, then look for the Decimal of 7 *d.* which is .00916667, and set that down also: Lastly, seek the Decimal of 2 *q.* which is .00208333, set that down also; then if you add these three numbers together, as in common Addition, you shall find the sum of them to be .68125000, which is the Decimal belonging to 13 *s.* 7 *d.* 2 *q.* as by the work in the Margine appeareth.

.65
.02916667
.00208333
————
.68125000

PROP.

PROP. II.

How by the Table *to expresse* Troy-weight *in Decimals.*

The second Table is of *Troy weight*, the several Denominations whereof are *Ounces*, *Peny-weights*, and *Grains* : So that by the Table you shall find that the Decimal belonging to five ounces is .41-*666667*, the Decimal belonging to 17 peny-weight is .. 07083333, and the Decimal belonging to 13 Grains is .00225694, and so of any other number of ounces, peny-weights, and grains severally.

But if it were required to expresse these (or any other) several Denominations in one Decimal Fraction, then you must (as before you did for money) take out of the Table the several Decimals belonging to the respective quantities, and add them together, so shall the sum of that Addition be the Decimal sought.

Example

If it were required to find a Decimal which should represent 5 ounces, 17 peny-weight, 13 grains, you must first look in the Table for the Decimal belonging to five ounces, which is .41666-*667*, and write it down, then look the Decimal belonging

longing to 17 grains, which is .07083333, and write that down, then look for the Decimal of 13 grains, which is .00225694, and write that down, then adding these three numbers together, you shall find the sum of them to be .48975-694, which is the Decimal representing 5 ounces, 17 grains, 13 peny-weight, as by the operation in the margent appeareth.

.41666667
.07083333
.00225694
.48975694

PROP. III.

How by the Table *to expresse* Avoirdupois great weight *in Decimals.*

The third Table is of *Avoirdupois* great weight, the several Denominations whereof are *Quarters* of *Hundreds*, *Pounds*, *Ounces*, and *Quarters* of *Ounces* thus you shall find in the Table, that the Decimal of 3 Quarters of a Hundred is .75, the Decimal of 22 pounds is .19642857, the Decimal of 7 ounces is .00390625, and the Decimal of 3 quarters, of an ounce is .00041853, in this manner by the Table you may find the correspondent Decimal belonging to any number of quarters, pounds, ounces, and parts of ounces severally.

But if it be required to find one Decimal number which shall represent divers denominations, you must first finde the Decimal belonging to the several particulars, and add them together,

gether, the sum whereof shall be the entire decimal required.

Example.

Let it be required to find a decimal which shall represent 3 quarters, 22 pounds, 7 ounces $\frac{3}{4}$ of an ounce. First, look in the Table for the decimal of three quarters of a hundred, which is .75, and write it down, then look for the decimal of 22 pound, which is .19642857, and write that down, also look the decimal belonging to seven ounces, which is .00390625, and write that down: Lastly, seek the decimal of three quarters of an ounce, which is .00041853, and write that down, then adding these four numbers together, you shall find their sum to be .95075335, which is the Decimal representing 3 qu. 22 *lib.* 7 oun. $\frac{3}{4}$ of an ounce.

.75
.19642847
.00390623
.00041853
.95075335

PROP. IV.

How by the Table to express Avoirdupois little weight *in Decimals.*

The fourth Table is of *Avoirdupois* little weight, the denominations whereof are *Ounces*, *Drams*, and *Quarters* of drams, so that the Decimal of 11 ounces is .6875, the Decimal of five drams is .01953125, and the decimal of one quarter of a dram is .00097656.

But if it be required to find one decimal number, which shall represent 11 ounces, $5\frac{1}{4}$, then you must first look for the decimal belonging to 11 ounces, which is .6875, and set it down, then look for the Decimal answering to 5 drams, which is .01953125, and set that down. Lastly, look the decimal belonging to a quarter of a dram, which is .00097656, and set that down; these three numbers being added together, produce .70800781, which is the correspondent Decimal belonging to 11 ounces, 5 drams, and a quarter of a dram.

.6875
.01953125
.00097656
.70800781

PROP. V.

How by the Table to express Liquid Measures in Decimals.

Because there is so great variety of Liquid measures, that hardly any two commodities are sold by the same, the difference of the gallon continually making alteration, we have therefore in this fifth Table made the greatest denomination to be one gallon, the next less denomination being pints and quarters of pints; so that in the Table you shall find the decimal belonging to three pints, to be .375, and the decimal belonging to two quarters, or half a pint, to be .062.5, and so for any other.

But for to express pints and parts of pints in one entire

entire decimal number, you muſt add the Decimals of the ſeveral denominations together, and their ſumme ſhall be the entire Decimal.

So if you were to expreſs 3 pints and an half in one entire decimal number, add the decimal of three pints, which is .375, to the decimal of two quarters, which is .0625. and their ſum .4375, ſhall be the Decimal of three pints and an half.	.375 .0625 ——— .4375

PROP. VI.

How by the Table to expreſſe dry meaſures in Decimals.

The ſixth Table is of *Dry Meaſures*, the ſeveral denominations whereof are *Buſhels, Pecks, quarters of Pecks* and *Pints*, ſo may you find the decimal of five buſhels to be .625, the decimal of two pecks to be .0625, the decimal of three quarters of a peck to be .023437, the decimal of two pints to be .0039063. Thus are the correſpondent decimals belonging to the ſeveral denominations found.

But if you would have one number to expreſs 5 buſhels, 2 pecks, three quarters of a peck, and 2 pints: you muſt firſt find the decimal belonging to 5 pecks, which is .625, and write it down, then find the decimal of two pecks, which is .0625, then ſeek the decimal of three quarters of a peck, which is .0234375, and write that down. Laſtly,

 ſeek

seek the decimal of two pints, which is .0039063, which numbers being added together, produce .7148438, which is the decimal belonging (or expressing) 5 bushels, 2 pecks, three quarters of a peck, and 2 pints.

.625
.0625
.0234375
.0039063
.7148438

PROP. VII.

How by the Table to express Long Measures *in Decimals.*

The seventh Table is of *Long Measures*, the Integers being *Yards* and *Ells*: and the lesser denominations are *quarters* of *Yards* or *Ells*, *Nailes*, and quarters of *Nailes*. So you may find in the Table that the decimal of three quarters of a Yard, or an Ell is .75, the decimal of two Nailes, is .125, and the decimal of one quarter of a Naile is 015625.

But if you would have one number to express 3 quarters of a Yard, or an Ell, two Nailes, and one quarter of a Naile, you must seek the decimal of three quarters of a Yard or Ell, which is .75, and write it down, likewise seek the decimal of two Nailes, which is .125, and write that down. Lastly, seek the decimal of one quarter of a Naile, which is .015625, and write that down, these three numbers added together, make .890625, which is the

.75
.125
.015625
.890625

decimal

decimal belonging to 3 quarters of a Yard or Ell, 2 Nailes, and one quarter of a Naile.

PROP. VIII.

How by the Table to express the parts of Time *in Decimals.*

Time is usually divided into *Years, Moneths* and *Dayes*: So the eighth Table which is of *Time*, consisteth of these two denominations, *Moneths* and *Dayes*, you may find that the decimal of 5 moneths is .416667, the decimal of 26 dayes is .071233. These are the principal decimals, but the compound decimal number representing 5 moneths, 26 days, is .487900, as you shall find, if you add .071233, which is the decimal of 26 dayes, to .416667, which is the decimal of 5 moneths.

PROP. IX.

How by the Table to express Dozens *in Decimals.*

The last Table is of *Dozens*, the Integer being a *Grosse*, and the smaller denominations are Dozens, and parts of Dozens, so may you find the decimal of seven dozen to be .5833333, and the decimal

of five parts of a dozen to be .0347222, and these two numbers added together, make .6220555, which is the number which representeth 7 dozen, and $\frac{5}{12}$ parts of a dozen.

In the setting down of Decimal Fractions, to add them together, you must alwayes observe to set *Primes* under *Primes*, *Seconds* under *Seconds*, &c. which the points before the several Fractions will direct you to do.

Hitherto we have shewed the use of the foregoing Tables in expressing of Fractions in decimal numbers. It resteth now to shew the use of them in finding what fraction either of Money, Weight or Measure, any decimal number given doth represent, and that shall be made evident by the ensuing Proposition.

PROP. X.

A decimal number being given, how to find what Fraction it doth represent.

Let .0291666.7 be a decimal number, representing some Fraction part of *English Coin*: Because it is required to find the value of this Fraction in *English Coin*, you must therefore repair to the Table

ble of *English Coin*; in the ſecond column of which Table ſeek for the number given (*viz.* 02916667) which you ſhall find to ſtand againſt 7 pence, and ſo much is the value of the decimal Fraction 02916667, in Engliſh Coin.

Alſo if the decimal Fraction .75 were given, you ſhall find the value thereof to be 15 ſhillings, and the value of .003125 to be three farthings.

Likewiſe in the Table of *Troy weight*, if .41666667 were given, it would ſignifie five ounces, and .05416667 would expreſs 13 peny weight, and .00173611 will expreſs ten grains, &c.

After this manner may you find the value of any decimal number given, either in *Money*, *Weight* or *Meaſure*, when the number given may be exactly found in the Table: But if the number given cannot be found exactly in the Table unto which it is directed at one entrance. Then you muſt, *Find in the ſame Table, the neareſt number you can leſs than the given number, and take the number that anſwers unto it in the firſt column, which will be the greateſt Fraction of the number required: then ſubſtracting the decimal thus found, out of the decimal given, you ſhall have a remainder, which remainder ſeek alſo in the ſecond column of the Table, if it may be found; if not, ſeek the neareſt leſſe, and the number anſwering thereunto in the firſt column ſhall be the next greateſt fraction; then ſubſtracting this decimal found out of the former remainder, there will be another remainder, which alſo ſeek in the Table, and proceed as in the former*: An Example or two will make all plain.

Example

Example 1.

Let .68125000 be a Decimal given, representing some part of *English Coin.* If you look in the Table of English Coin for .68126000, you cannot find it, but the nearest number in the Table lesse than it is.65, against which I find .13 *s.* so that 13 *s.* is the greatest fraction part of *English Coin* agreeing to this number.

This done, substract .65 out of .68125000, and there will remain .03125000, which number also you must seek in the Table of *English Coin*, but being you cannot find it there, you must take the nearest number less than it, which is .02916667, against which I find 7 pence, which is the next greatest fraction part of *English Coin* agreeing to this number.

Again, substract .02916667, out of .03125000, and there will remain .00208333, which number seek in the Table, and you shall find it to stand against 2 farthings, and so much doth this last remainder signifie in *English Coin*, and the whole given number .68125000 doth represent in *English Coin* 13 shillings 7 pence 2 farthings, as by the operation following doth appear.

.68125000

.68125000 number given.
.65 . . . . . . the next lesser number in the Table representing 13 *s.*

.03125000 first remainder.
.02916667 the next lesser number in the Table representing 7 *d.*

.00208333 second remainder, which represents two farthings.

So doth the whole number represent 13 *s.* 7 *d.* 2 *q.*

Example 2.

Let the Decimal .87426934 representing some fraction of a pound sterling be given. If you look in the Table of *English Coin* for .87426934 you cannot find it; but the nearest number in the Table less than it is .85, against which I find 17 shillings, so that 17 shillings is the greatest fraction part of *English Coin*, agreeing to this number.

Then substracting .85 out of .87426934, there will remain .02426934, which number also you must seek in the Table of *English Coin*, but seeing you cannot find it there, you must take the nearest number lesse than it, which is .02083333, against which I find 5 pence, which is the next greatest fraction part of *English Coin*.

Lastly, substract .02083333, out of .02426934, and there will remain .00343601, which number you must also seek in the Table of *English Coin*; but not finding it exactly there, you must take the nearest number lesse, which is .003125, against which

which you shall find 3 farthings, which is the next greatest fraction part of *English Coin*, and the Decimal .87426934, doth in value signifie 17 shillings 5 pence 3 farthings, and something more, for .003125 is the decimal of 3 farthings; and the number you are to look for in the Table is .00343601, greater than the decimal of 3 farthings; wherefore, if you substract .003125 out of .00343601, there will remain .31101, which is the $\frac{31101}{100000}$ part of a farthing, which is inconsiderable. See the following operation.

87426934 Decimal given.
85 . . . . . . Decimal of———— 17 *s.*

92426934 First remainder.
02083333 Decimal of———— 5 *d.*

00343600 Second remainder.
003125 . . Decimal of—— 3 *q.*

00031101 Decimal part of a Farthing.

¶ And here note, that whatsoever hath been here said concerning the uses of the Table of English Coin, the same order is to be observed in the use of the other Tables of *Weight, Measure, Time,* &c. as by the following Examples (if you make trial) will appear.

Examples.

Examples.

1 If this decimal 48975694, were given to know the value thereof in *Troy weight*, you ſhall find it to contain 15 ounces, 17 peny-weights, and 13 grains.

2 Alſo if .95075335 were a Decimal given, and it were required to find the value thereof in *Avoirdupois great weight*, you ſhall find it to contain 3 quarters of a hundred, 22 pound, 7 ounces, and 3 quarters of an ounce.

3 Likewiſe, if .70800781 were a decimal Fraction given, you ſhall find the value thereof in *Avoirdupois little weight* to be 11 ounces, 5 drams, and one quarter of a dram.

4 If .4375 were a Decimal, whoſe value were required in *Liquid Meaſure*, you ſhall find it to contain 3 pints and an half.

5 Let .7148438 be a Decimal given, whoſe value is required in Dry meaſures, you ſhall find it to contain 5 buſhels, 2 pecks, 3 quarters of a peck, and 2 pints.

Thus have I ſhewed you the uſe of theſe decimal Tables in expreſſing of the fraction parts of *Money*, *Weight*, *Meaſure*, &c. But becauſe theſe Tables may not be alwayes at hand, when there is need of them; I will here ſhew you how the value of any Decimal given, may be known by multiplication onely; and this is

THE

THE RULE.

Multiply the Decimal *given, by the number of known parts of the next inferiour* Denomination, *which are equal to the* Integer, *the* Product *is the value of the* Decimal *proposed in that inferiour* Denomination; *and if there happen to be any* Decimal *in the* Product, *you may in like manner find the value thereof in the next inferiour* Denomination, *and so proceed till you come to the least known parts of the Integer.*

Example.

Let .67395834 be a Decimal given, representing the fraction of a *Pound sterling.* First, multiply .67395834 by 20, (the number of shillings in a pound sterling) and the Product will be 1347916680, from which cutting off the last eight figures with a point, or dash of the pen (because there were eight figures in the given Fraction) there will stand before the point (towards the left hand) 13, which are shillings, and the remainder .47916680 standing behind the point, will be the fraction part of one shilling sterling, which number .47916680, you must multiply by 12 (the number of pence in one shilling) and the Product will be 575000160, from which number cut off the last eight figures as before, and there will be 5 left to the left hand, which are 5 pence, and the figures on the right hand of the point, *viz.* .75000160 are the fraction part of one peny sterling, which therefore multiply by 4 (the number of farthings in one peny)

peny) and the Product of that multiplication will be 300000640, from which cut off the last eight figures to the right hand, and there will be left 3 towards the left hand, which representeth 3 farthings, and the remaining figures towards the right hand are but the fraction part of a farthing, which we therefore reject. And thus you find by *Multiplication* only, that this fraction .67395834 doth represent in the known parts of *English Coin*, 13 shillings 5 pence 3 farthings, as by the following operation appeareth.

	.67395834
	20
Shillings	13,47916680
	12
	95833360
	47916680
Pence	5,75000160
	4
Farthings	3,00000640

In like manner, if this fraction .94809028 were given, representing some fraction part of Troy weight, you shall find the value thereof to be 11 Ounces, 7 peny weight, 13 grains, as by the operation following appeareth.

.94809028

```
               .94809028
                      12
               ---------
               189618056
               94809028
               ---------
Ounces.      11,37708336
                      20
               ---------
Peny-weight.   7,54166720
                      24
               ---------
                216666880
               208333440
               ---------
Grains.        1310000128o
```

In this manner may any Decimal given be reduced into the known parts of the Integer by *Multiplication* only. And

¶ Here note, that whereas in the preceding Tables the decimal fractions consist of *seven* or *eight* Figures, we shall in the prosecution of our work make use onely of *four* or *five* of the first of them, which will be sufficient in ordinary practice, and come near enough to the truth in any ordinary question whatsoever.

So if in stead of .02916667, which is the fraction part of 7 pence, you take out only .02916, it will be sufficient.

Also

Also for	.05833333	take	.05833	in *Troy weight*
	.0058594		.0058	in dry measure
	.5833333		.5833	in Time.

Thus much concerning the construction and use of the decimal *Tables*, we shall now come to the practice of *Decimal Arithmetick*, which shall be taught in the Rules following.

Of Notation of Decimals.

NOTATION of Decimals is contrary to that of whole numbers: for whereas in whole numbers the values of figures are increased ten-fold by continual addition of Cyphers towards the right hand: so, on the contrary, the values of the places of Decimals do decrease in the same proportion.

And whereas in whole number, Cyphers in the first place towards the left hand are unnecessary, yet in Decimals they are absolutely necessary to discover the true denominator. Also Cyphers at the end (or towards the right hand) of decimal numbers are of no value, for one single Figure in decimals signifies as much as the same Figure would do, if there were Cyphers placed behind it, so 7 is equivalent unto 70, 700, or 7000, &c. For the denominators of decimal Fractions are alwayes Cyphers with a unite to-

wards the left hand, as hath been already intimated. So $\frac{70}{100}$ being reduced to its least terms will be $\frac{7}{10}$, & $\frac{7000}{10000}$ will be reduced to $\frac{7}{10}$ also, and so of any other, as by the Table following doth evidently appear.

987654321	12345678
100000000	.00000001
10000000	.0000001
1000000	.000001
100000	.00001
10000	.0001
a thousand 1000	.001 or $\frac{1}{1000}$
a hundred 100	.01 or $\frac{1}{100}$
Ten 10	.1 or $\frac{1}{10}$

Addition of Decimals.

IN Addition of Decimals, the same order is to be observed as in Addition of numbers of one denomination before taught in the first part, in which there is no difficulty : But in Decimal numbers the chief care to be taken is in placing your whole numbers and Fractions in their due order, which you shall easily and certainly do, if you observe this general rule, *viz.* to *place your whole numbers and fractions one under another, so that the points of separation which* (in decimal numbers) *distinguish the whole numbers from the fractions, stand directly one under the other,* then are you to proceed in

in the addition of them in all respects, as you did in whole numbers.

Example 1.

Let it be required to add together in one sum these several sums following, in a decimal way, *viz.* 36 *li.* 2 *s.* 8 *d.* 29 *li.* 0 *s.* 2 *d.* 21 *li.* 16 *s.* 9 *d.* and 6 *li.* 2 *s.* 5 *d.*

First, set down 36 *li.* and a point or Comma after it, then for the fraction part of 2 *s.* 8 *d.* look in your Table of English Coin, where you shall find decimal fraction of 2 *s.* 8 *d.* to be, . 1333 therefore for 36 *li.* 2 *s.* 8 *d.* set down 36. 1333.

Secondly, for your 29 *li.* 0 *s.* 2 *d.* set down 29. 0083.

Thirdly, for your 31 *li.* 16 *s.* 9 *d.* set down 31. 8375.

Lastly, for your 6 *li.* 2 *s.* 5 *d.* set down 6. 1208 as you see done in the operation following.

	li.	*s.*	*d.*		
For	36	02	8	set down	36, 1333
	29	08	2		29, 0083
	31	16	9		31, 8375
	6	02	5		6, 1208
	103	02	0		103, 0999

Your decimal numbers being thus placed in due order one under another, proceed to the adding of them together, as if they were whole numbers, and you shall find the sum or total of them to be 103. 0999.

Now the 103 which ſtands towards the left hand, are 103, pounds, and the .*0999* which ſtands towards the right hand of the Comma, is the fraction part of one pound ſterling, the value whereof you may find (by the Propoſition before going) to be two ſhillings *fere*, which ſhould be two ſhillings exact, but it wanteth ſomewhat, *viz.* the $\frac{2}{1000}$ part of a farthing, which is inſenſible; for if by the fore mentioned rule you ſeek the value of the decimal fraction, .*0999*, you ſhall find it to be 1 ſhilling, 11 pence, 3 farthings, and the $\frac{2}{1000}$ part of a farthing, which you may call in all 2 ſhillings, for decimal numbers will ſeldome happen to give the exact value of fractions, but will be either greater or leſſer than they ought to be; but in ſuch a ſum as this is, the thouſand part of a farthing is not to be regarded.

Example 2.

Let it be required to add together in a decimal way theſe ſums following, *viz.* 29 *li.* 18 *s.* 7 *d.* 3 *q.* 63 *li.* 11 *s.* 2 *d.* 1 *q.* 129 *li.* 4 *s.* 0 *d.* 2 *q.* and 3 *li.* 7 *s.* 10 *d.* 1 *q.*

Firſt, for 29 *li.* 18 *s.* 7 *d.* 3 *q.* ſet down 29. *93229.*

Thirdly, for 129 *li.* 4 *s.* 2 *q.* ſet down 129.20208

Laſtly, for 3 *li.* 7 *s.* 10 *d.* 1 *q.* ſet down 3.39271 as you ſee here down in the Margine.

Your

Your decimal numbers thus placed in order, add them together, as if they were whole numbers, and you shall find the sum of them to contain 226.08645.

29.93229
63.55937
129.20208
3.39371
226.08645

Now the 226 which stands towards the left hand of the Comma, are 226 pounds, and the other figures towards the right hand, *viz.* 08645 are the Fraction parts of a pound sterling, which if you reduce by the fore-mentioned Proposition, you shall find the value thereof to be 1 shilling, 8 pence, 3 farthings, so the whole sum is 226 *li.* 1 *s.* 8 *d.* 3 *q.*

And here note, that what hath been said, as concerning *Money*, the same is also to be understood of *Weight*, *Measure*, *Time*, &c. as by the following Examples will appear.

Other Examples for Practice.

Example 1	*Example* 2.
In Money.	In Troy weight
135.8833	7.97413
95.5583	6.65330
3.2875	3.62187
234.7291	18.24930
234 *li.* 14 *s.* 7 *d.*	18 *li.* 2 *ou.* 19 *p. w.* 20 *gr.*

Example 3. In *Avoirdupois little w.*	*Example* 4. In *Avoirdap. great weight.*
12.7227	37.9442
76.3594	9.3053
32.625	33.6786
91.4883	10.0000
32.8398	12.8142
246.0398	103.7423
246 *li.* 00 *oun.* 9 *dr.*	103 *C.* 2 *q.* 27 *lb.* 3 *oun.*

Substraction of Decimals.

THe Substraction of Decimals differeth nothing from the substracting of one whole number from another, and the decimal numbers to be substracted one from another, must be placed in the same order, as in *Addition* of Decimal numbers, the practice of Substraction shall be seen in the following Examples.

Example 1.

Let it be required to substract 31 *li.* 16 *s.* 9 *d.* out of 36 *li.* 2 *s.* 8 *d.*

First, for your 36 *li.* 2 *s.* 8 *d.* set down the decimal thereof, which is 36. 1333.

Secondly, for your 31 *li.* 16, *s.* 9 *d.* set down the Decimal thereof 31. 8375.

This

This done draw a line under them, & substracting the lesser from the greater, you shall find the remainer to be 4.2958 the 4 on the left side of the Comma are four pounds, and the .2958 which standeth towards the right hand, is the fraction part of a pound, the value whereof being sought, will be found to be 5 *s.* and 11 pence. So that if you substract 31 *li.* 16 *s.* 9 *d.* out of 36 *li.* 2 *s.* 8 *d.* there will remain 4 *li.* 5 *s.* 11 *d.*

36, 1333
21, 8374
4, 2958

But if divers sums be to be substracted out of one greater sum, then you must first add all the several smaller sums together, and substract the sum of them from the greater given sum, so shall the remainder be the sum desired.

Examples for Practice.

Example 1.

In Money.

Lent	2784.8375
Paid at several times	36.1333
	29.0083
	31.8375
	6.1208
paid in all	103.0999
rests to	2581.7376
pay	2581 *li.* 14 *s.* 0 *d.*

Example 2.

In *Avoirdupois* great weight.

Bought	103.7423
Sold	37.9442
Unsold	65.7981
65 *C.* 3 *q.*	5 *l.* 7 *oun.*

Example 2.

in Troy weight

Delivered to a Goldsmith of old Plate	7.97413
Received of new Plate	5.59670
Rests in the Goldsmiths hands	2.7743
	2 *li.* 4 *oun.* 10 *p.w.* 14 *gr.*

Multiplication of Decimals.

MULTIPLICATION *of Decimals* differeth nothing at all from the *Multiplication* of whole numbers, for making the greater number the *Multiplicand*, and the lesser number the *Multiplier*, the number issuing from that multiplication shall be called the *Product*.

Now in the multiplication of decimal numbers one by another, if there be any Fraction either in the *multiplicand* or *multiplier*, or Fractions in both: So many figures as the Fractions contein, so many figures must be cut off from the *Product* towards the right hand, which shall be the Fraction of the *Product*, and the figures towards the left hand of the Comma in the Product, shall be the Integers of the Product.

Example 1.

Let it be required to multiply 34 pounds, five shillings, three pence, by 16 pounds, six shillings, six pence.

First, seek the Decimal of 34 *li.* 5 *s.* 3 *d.* which you shall find to be 34.2625, make this number your Multiplicand, then seek the Decimal of 16 *li.*

6 *s.*

6 s. 6 d. which you ſhall find to be 16, 325, make this decimal number your Multiplier, then draw a line, and multiply theſe two numbers together, as if they were whole numbers, and you ſhall find the Product of them to be 559.3353125. Now becauſe there are four figures in the Multiplicand which are Fractions, namely, theſe four towards the right hand, *viz.* 2625, and there are alſo three figures in the multiplyer, which are Fractions, namely, theſe three towards the right hand, *viz.* 325, that is in all ſeven figures, repreſenting Fractions I therefore cut off from the product the ſeven figures towards the right hand, by making of a Comma there, to diſtinguiſh the whole number from the fraction : So is 559 the Integer or whole number, and .3353125, the Fraction of this multiplication.

Multiplicand	34.2625
Multiplier	16. 325
	1713125
	685250
	1027875
	2055750
	342625
Product	559.3353125

Example 2.

If there be Fractions in the multiplicand, and none in the multiplier, yet the work is ſtill the ſame, for you muſt cut off only ſo many figures from the product, as there are Fractions either in multiplicand, multiplier, or both : So if it were required to multiply 5767 yards, and 3 quarters of a yard, by 235 yards, you muſt firſt ſet down 5767.75 for your

your 5767 yards, and three quarters, which number must be your multiplicand: Also set down 235 yards for your Multiplier, then multiplying them together, as if they were whole numbers you shall find the product to be 1355421.25, and because there are only two Fraction figures, both which are in the multiplicand, namely, the two last thereof .75, and none in the multiplier. I therefore cut off only two figures of the product, namely, the two last, which are .25, so is the product of this multiplication 1555421.25 which is 1355421 square yards, and one quarter of a yard. And so if a garden or other piece of land lying square should contain in length 5767 yards, and three quarters, and in breadth 235 yards, the whole piece would contein 1355421 square yards, and one quarter of a yard.

```
   5767,75
       235
  --------
   2883875
  1730325
 1153550
----------
1355421,25
```

Example 3.

If decimal Fractions be to be multiplied by decimal Fractions, you must then (as before) multiply them as whole numbers, and from the Product cut off so many Figures towards the right hand, as there are Figures in the multiplicand and the multiplier: So if it were required to multiply .953 by .782, you

shall

ſhall find their product to be .745246, which being but 6 figures in all I cut them off and that fraction .745246 is the product of the multiplication of the two given fractions.	.953 .782 —— 1906 624 6671 —— .745246

Example 4.

If any two Decimal fractions being multiplied together, the product thereof doth not conſiſt of ſo many places as are required (by the former rules) to be cut off, you muſt then ſupply that defect by prefixing a Cypher, or Cyphers before the product towards the left hand: So if theſe Decimal fractions

.063 and .0752 were to be multiplied, their product would be 47376. Now (by the former rules) you ſhould cut off ſeven figures of the product towards the right hand, but this product 47-376 conſiſteth but of five figures; wherefore to make	.0752 .063 —— 2256 4512 —— .0047376

it ſeven figures, I prefix two Cyphers before the product on the left hand, making it .0047376, and that is the true product produced by this multiplication.

Example

Example 3.

If you would multiply any Decimal (either Fraction only, or whole number and fraction together) by 10, 100, 1000, *&c.* You must add so many Cyphers to the multiplicand, as there are Cyphers in the multiplyer, and cut off so many Figures as there are fractions in the multiplicand, and that number shall be the product required: So if 7, 856025 were a Decimal given to be multiplied by 100, add two Cyphers to the number given, making it 785602500, then because there were six figures of this number towards the right hand, it will be 785,602500, which is the true product required.

Examples for Practice.

Example 1.	*Example* 2.
7,432	22,358
2,61	- 32
7432	44716
44592	67074
14864	715.456
1939752	

Example

Example 3.

```
 .352
  .24
-----
 1408
 604
-----
.7448
```

Example 4.

```
375.6218
     100
----------
375.621800
```

Division of Decimals.

AS *Division* of whole numbers is the hardest of the four *Species* of *Vulgar Arithmetick*, so the *Division* of *Decimals* is the most difficult of the four kinds of *Decimal Arithmetick*, but I hope to make it plain, to the understanding of the meanest capacity.

The several varieties that may happen in Division, are principally (if not only these) *four*. Namely, First, *To divide whole numbers and fractions, by whole numbers and fractions.* Secondly, *To divide whole numbers by mixt, or mixt numbers by whole.* Thirdly, *To divide a greater fraction by a less*; and Lastly, *To divide a lesser fraction by a greater.*

In Division of Decimals this Rule is general, *If the* dividend *be greater than the* Divisor, *the* quotient *will be either a* whole number *or* a mixt, *but if the* Dividend *be less than the* Divisor, *the quotient will*

will be a Decimal. And (for convenience in working, if there be need) any number of Cyphers may be annexed to the Dividend, that thereby the quotient may extend to as many places as the tenour of the question ſhall require.

The manner of the working of *Diviſion* in Decimals, is the ſame with that before delivered in whole numbers in the firſt part of Vulgar Arithmetick, as will appear by the Examples following, in every of the four premiſed varieties.

The Rule for the firſt variety.

The Dividend *and the* Diviſor, *being both* mixt numbers, *or one of them being a* whole number *and the other* a mixt; *or the* Dividend *being a* Decimal, *and the* Diviſor *a* whole number *or* a mixt, *the firſt figure in the quotient will be of the ſame* place *or* degree, *with that figure or Cypher of the* Dividend, *which at the firſt demand ſtandeth, or (at leaſt) is ſuppoſed to ſtand directly over the place of* Unites *in the* Diviſor.

Example 1. *Where the terms given are both mixt numbers.*

Let it be required to divide 659.3354125 by 16.325. Here the terms given are both of mixt numbers, which being placed according to the Rules delivered before, for the Diviſion of whole numbers, the figure in the Dividend, which at the firſt demand, ſtandeth over 6, the place of Unites in the Diviſor is 5, and becauſe this ſtandeth in the place

of

of tenths, therefore the first figure in the quotient is in the place of tenths also, and the whole number consisteth of two of the foremost places, and the rest is a Decimal, thus the quotient sought in our present example is 34.2625, of which 34 the two first figures is the Integer or whole number, 2625 the Decimal fraction.

Divisor	*Dividend*	*Quotient.*
16.325)	559.3353125	(34.2625
	......	
	489.75	
	69.585	
	65.300	
	4.2853	
	3.2650	
	1.02031	
	97950	
	040812	
	32650	
	081625	
	81625	

Example 2. One of the terms given, being a whole number, the other mixt.

The mixt number 137542 1.26 being divided by

by the whole number 235, the quotient will be 5767.75, and the first figure in the place of Thousands, as by the operation it doth appear.

Divisor	*Dividend*	*Quotient*
235.)	135542125	(5767.75
	1175	
	1804	
	1645	
	1592	
	1410	
	1821	
	1645	
	1762	
	1645	
	1175	
	1175	

Example 3. The Dividend being a Decimal, and Divisor a whole number.

The Decimal fraction .35673 being divided by the whole number 26, the quotient will be .001372, and the first significant figure in the place of thousands, or fourth place from Unity, as by the operation it doth appear.

The

```
26) .35673 (001372
      ...

    ______
    26
    096
    ______
     78
     187
    ______
     182
     0053
    ______
       52
```

The Rule for the second variety.

When the Dividend *is a* whole *or* mixt *number, and the* Divisor *a* Decimal, *add as many Cyphers to the* Dividend *as there are places in the* Divisor *for the integral part of the quotient will consist of as many places as the* Divisor, *and the places arising from the integral parts of the* Dividend *added together.*

Example 1.

Let 348.75 be the mixt number given, to be divided by the decimal .25, to the number given, I add to Cyphers, the number of places in the divisor, and then it will be 348.7500, which being divided by .25 ; the integral part of the quotient will be 1395. because the whole part of the dividend 348, being divided by .25 giveth two places, and

the number of places in the Divisor being two, giveth two more; and so the Integral part consisteth of four figures, as by the operation.

Divisor	*Dividend*
.25)	348.7500 (1395
	25
	98
	75
	237
	225
	125
	125

Example 2.

Let the mixt number 72.5 be divided by .075, the number of places in the integral part of this Quotient will be *966*, because there are 3 places in the Divisor; and but 3, because the inte-

Divisor	*Dividend*
075)	72.5000 (966
	675
	500
	450
	500
	450
	500
	450

gral

gral part of the dividend is lesse than the significant figures in the Divisor, as by the operation it doth appear.

The Rule for the third variety.

When the Terms given are both Decimals, *the* Dividend *being the* greater, *the integral part of the quotient will consist of as many places as the Divisor doth.*

Example.

Let the Decimal .73658 be divided by the Decimal .32 the integral part of the quotient will be 23, because the Divisor doth consist of two places, as by the operation in the margine it doth appear.

```
Divisor   Dividend
  .32)    .73958 (23.11
           ....
          -------
          64
           99
          -------
           96
            35
          -------
            32
             38
          -------
             32
              6
```

The Rule for the fourth variety.

When the terms given are both decimals, *consisting of equal places, the* dividend *being the* lesser

 term,

term, *place the* dividend *as a* Numerator, *and the Divisor as* denominator; *so is such vulgar fraction the quotient sought: But if the terms given consist not of equal places, supply the place or places wanting in either of the terms, by annexing a Cypher or Cyphers on the right hand, and then proceed as before.* Thus if .27 be given to be divided by .93, the quotient will be $\frac{27}{93}$. Also if .35 be given to be divided by .78563, the quotient by annexing 3 Cyphers to .35, the lesser decimal given will be $\frac{35000}{78563}$, which vulgar fractions may be reduced into decimals if need be, by the first Proposition in this Second part of decimal Arithmetick.

Examples for Practice.

```
       . . .
44) .35673 (.0081,&c.        .25) 2481.00 (9924
     352                           225
      47                           0231
      44                            225
       3                             60
                                     50
                                    100
                                    100
                                    000
```

The

The Rule of Three in Fractions *Vulgar* and *Decimal*.

WHat the Rule of Three is, and the manner of working, is already ſhewed in the firſt part, that which we here intend is only to add ſome Examples in fractions vulgar as well as decimal; that by comparing the work in both, the excellent uſe of decimal Arithmetick might the better appear.

And how to convert the known parts of *Money*, *Weight*, or *Meaſures Engliſh*, into *Decimals* hath been already ſhewed, both *Arithmetically* and by *Tables*; yet to prevent the ſeveral *additions* and *ſubſtractions* in thoſe *Tables*, I have here annexed another *Decimal Table*, for the more ſpeedy Reduction of *Engliſh money* under two ſhillings, all ſummes of money above, not having pence or farthings annexed, being as eaſily reduced by *memory* as by *Tables*; and this I have the rather done, becauſe the ſame *Table* will alſo reduce the *Coins* of *France*, and the parts of *Troy-weight*, if an ounce be made the Integer, which in point of practice is much more uſeful then the pound.

The Table of Reduction.

Peny	001041			026041	
	002083	Gr. 1		027083	13
	003125			028125	
1	004166	2		029166	14
	005208			030208	
	006250	3		031250	15
	007291			032291	
	008333	4	8	033333	16
	009375			034375	
	010416	5		035416	17
	011458			036458	
3	012500	6		037500	18
	013541			038541	
	014583	7		039583	19
	015625			040625	
4	016666	8		041666	20
	017708			042708	
	018750	9		043750	21
	019791			044791	
	020833			045833	22
	021875			046875	
	022916	11		047916	23
	023958			048958	
6	025000	12	12	050000	24

The Table of Reduction.

	051042			076014	
	052083	G. 1		077083	13
s.d.	053125			078125	
1. 1	054166	2	7	079166	14
	055208			080208	
	056250	3		081250	15
	057292			082921	
2	058333	4	8	083333	16
	059375			084375	
	060416	5		085416	17
	061458			086458	
3	062500	6	9	087500	18
	063542			088541	
	064583	7		089583	19
	065625			090625	
4	066666	8	10	091666	20
	067708			092708	
	068750	9		093750	21
	069792			094791	
5	070833	10	11	095833	22
	071874			098675	
	072916	11		099716	23
	073958			099858	
6	075000	12	12	000000	24

These things premised; we will now shew the use of the Table in some practical questions belonging to the Rule of three direct.

1 *Question.*

If $\frac{7}{8}$ of a yard of Cloth, cost $\frac{9}{12}$ of a pound: what shall 17 yards cost at the same rate?

If $\frac{7}{8}$ cost $\frac{9}{12}$, what shall 17 cost? Ans. 14 *li.* $\frac{4}{7}$.

First, multiply $\frac{9}{12}$ by $\frac{17}{1}$ the product is $\frac{153}{12}$, then divide $\frac{153}{12}$ by $\frac{7}{8}$, the quotient is $\frac{1224}{84}$: again, if you divide 1224 by 84, the quotient is 14 $\frac{48}{84}$, or in the least terms 14 pound $\frac{4}{7}$ of a pound.

And the value of this fraction $\frac{4}{7}$ of a pound, will be found by the third Rule of Reduction of Fractions to be 11 shillings 5 pence, and $\frac{4}{7}$ of a peny, which is somewhat above two farthings: for it is 2 farthings, and $\frac{2}{7}$ of a farthing.

The same Question in Decimals.

If $\frac{7}{8}$ of a yard of Cloth cost $\frac{9}{12}$ of a pound, what shall 17 yards cost at the same rate?

To answer this question $\frac{7}{8}$ of a yard, and $\frac{9}{12}$ of a pound must first be reduced into decimals, either by Division, or by the Tables of Reduction: by both which wayes of Reduction the decimal of $\frac{7}{8}$ will be .875, and the decimals of $\frac{9}{12}$ will be .75, and then the terms of the question will stand thus;

If .875 parts of a yard cost .75 parts of a pound, what shall 17 yards cost at the same rate?

If 0.875——0.75——17. Here if you multiply the second term 0.75 by 17 the third term given, the product will be 12.75. and this product divided

divided by .875, gives in the quotient 14.57142, that is 14 pound .57142 parts of a pound, or 145 Decades, that is 14 pound 10 shillings, and .7142 parts of a Decade (or two shillings) which by the preceding Tables is 1 *s.* 5 *d.* 2 farthings, and .0059 parts of a farthing.

2 *Question.*

If a piece of Gold plate weighing 19 ounces 3 peny weight and 5 grains, be worth 62 pounds 10 shillings 6 pence; what is one ounce of the same gold worth?

This question in vulgar fractions must be expressed thus.

If 1 *l.* $\frac{3437}{5760}$ Troy-weight, cost 62 *l.* $\frac{126}{240}$, what shall $\frac{1}{12}$ of a pound Troy cost at the same rate?

To answer this question the fractions 1 *l.* $\frac{3437}{5760}$ and 62 *l.* $\frac{126}{240}$, must be first reduced into improper fractions, and the fraction $\frac{1}{12}$ into the least known parts of a pound Troy, and then the question will stand thus.

If $\frac{9197}{5760}$ give $\frac{15006}{240}$, what shall $\frac{480}{5760}$ give?

Now because it is necessary the terms given be reduced into their least Denominations, before the question be resolved, therefore the answer may be found, by using the terms given thus reduced as whole numbers, not having any regard to the Denominators of these fractions; Saying thus,

If 9197 grains, cost 15006 pence, what shall 480 grains cost.

And here if you multiply 15006 by 480, the product will be 7202880, which being divided by 9197, the quotient will be 783 pence $\frac{1629}{9197}$ parts of

a

a peny; and dividing 783 by 12, it will be 65 shillings 3 pence $\frac{1619}{9197}$, or 3 *l.* 5 *s* 3 *d.* $\frac{1619}{9197}$. And although this question is thus more easily answered then it would have been, if the terms had been wrought as vulgar fractions, yet the same terms being reduced to Decimals, the answer of the question will yet be found with more ease, as shall appear by the operation following.

The same question in Decimals.

If a piece of gold plate weighing 19 *ounces* 3 *peny weight and* 5 *grains, be worth* 26 l. 10 s. 6 d. *what is an ounce of the same gold worth?*

The Decimal of 19 ounces 3 peny weight and 5 grains, making an ounce the Integer, is by this Table 19.16041, for that 19 ounces are 19 Integers, 2 peny weight is one tenth of an ounce, and the Decimal of one peny weight 5 grains is by this Table .06041; and the Decimal of 62 *l.* 10 *s.* 6 *d.* by the same Table is 62.525, and because an Unite or Integer is the third term given, there needs no multiplication, if therefore you divide 62.525 the second term, by 16.16041 the first term propounded, the quotient will be 3.2632, that is 3 pounds 5 shillings 3 pence, and somewhat more, as by the operation in the margine it doth appear.

19.16041)

19.16041) 62.5250000 (3.2652

57.48123
5.643770

3.832082
1.2116880

1.1496246
6206349

5748123
4582170

3832082
750088

3 *Question.*

If 5 Ells and a quarter of linnen Cloth cost 2 l. 16 s. 8 d. 3 q. *what shall* 278 *ells and a half cost at the same rate ?*

If you would work this Question by whole numbers, your easiest way is first to reduce all the terms into their least Denominations, that is to say, the ells into quarters, and the pounds, shillings, pence, and farthings all into farthings, so shall your 5 ells and a quarter be 21 quarters, and your 278 ells and an half will be 1114 quarters, and your 2 *l.* 26 *s.* 8 *d.* 3 *q.* will be 2723 farthings, and then will your question stand thus in whole numbers.

quarters *farthings* *quarters*

If 21—cost—2723— what will—1114 cost.

Then multiplying the second number by the third, that is, 2723 by 1114, the product will be

3033422,

3033422, which divided by 21, the quotient will be 144448 farthings, which being again reduced into pounds, shillings and pence, giveth 150 *l.* 9 *s.* and 4 pence, as by the operation following doth appear.

The Operation.

	li.	*s.*	*d.*	*q.*	
$5\frac{1}{4}$	2	16	8	3	$278\frac{1}{2}$
21	20				4
	40				1112
	16 *shillings*				2
	56				1114
	12				
	112				
	56				
	672				
	8 *pence.*				
	680				
	4				
	2720				
	3 *farthings.*				
	2723				
	1114				
	10892				
	2723				
	2723				
	2723				
	3033422				

21)

```
21) 3033422 (144448
    • • • • • • •
  ---------------
    21
     93
    ---
     84
      93
     ---
      84
       94
      ---
       84
       102
       ---
        84
        182
       ----
        168
         14
```

```
                      d.
 z              z (4     r     s.  d.
 x44448     (36xxz   (300  (9 (150
  44444      xzzzz    zzz   0
               xxx
```

But if you would work the ſame queſtion by Decimal numbers, you may ſave the labour of reducing the terms to their leaſt Denominations, for 5 Ells and a quarter is in decimal numbers 5.25, and 278 Ells and an half is 278.5, and 2 pound 16 *s.* 8 *d.*

8 *d*. 3 *q*. is in decimals 2.8364, and then your question in decimals, will stand thus.

Ells   *pounds*   *Ells*
If 5.25 cost 2.836, what 278.5

If you multiply (according to the Rule) the second term by the third, that is 2.8364 by 278.5, the product of that multiplication will be 789.93400, which divided by the first term 5.25, the quotient will be 150.4642, which decimal representeth 150 *l*. 9 *s*. 4 pence, and so much in money will 278 Ells and a half cost.

The Operation.

```
Ells        pounds         Ells
5.25 ———————— 2.8364 ———— 278.5
                27.85
              ———————
                14182
               226912
              198548
              56728
          ————————————
            789.93740
```

5.25) 789.937400 (150.4642
or
(150 *li.* 9 *s.* 4 *d.*

```
525
2649
----
2625
 2437
 ----
 2100
  3374
  ----
  3150
   2240
   ----
   2100
    1400
    ----
    1050
     350
```

I have been the larger in this Rule, and especially in this Example, which is incumbred with fractions sufficient, because I would have the Reader the better discern the difference between the *Vulgar* and the *Decimal* way, and also to see how expeditious the one is over the other. Now this example being thus largely explained, I shall with the more brevity pass over the Rules following, giving one Example, or two at the most in each Rule; And thus much shall suffice for the Golden Rule, or Rule of Three direct in Fractions.

An

The Rule of Three Reverse.

A *Lends* B. 233 l. 6 s. 8 d. *for a year without Interest, upon condition that* B. *should do the like courtesie for* A. *when required.* A. *hath occasion for money* 7 *moneths; how much money ought* B. *to lend* A, *to requite his courtesie, and save himself harmlesse?*

I will not in this place tell you what the Rule of Three reverse is, nor the manner of working thereof, that being already sufficiently declared in the first part, but give you the Example, and the working thereof, which take as followeth: So will the Question be thus stated.

moneths	*li.*	*s.*	*d.*	*moneths*
12	233	6	8	7

Which in Decimals stands thus,

moneths	*li.*	*moneths*
12	233.33	7

```
       233.33
           12
       ------
       46666
      23333
      ------
      2799.96
      6666(3
     x79996 (39999 1/4
      77777
```

Here

Here you ſee that 12 moneths and 7 moneths are whole numbers, and ſo we let them alone without any reduction, but the Decimal of 233 *l.* 6 *s.* 8 *d.* will be found by the fore-mentioned Tables and Rules to be 233.33, which is the middle term in the queſtion, and of the ſame quality with that, muſt the fourth term ſought be, therefore if (according to the Rule delivered in the firſt part) you multiply 233.33 by 12, the product will be 2799.96, which divided by 7, giveth in the quotient 39999, which is the Decimal of 400 *l.* and ſo much money ought B. to lend A. for 7 moneths.

The Rule of Proportion, conſiſting of five Numbers.

Queſtion 1.

If 100 li. *in* 12 *moneths yields* 6 li. *intereſt, what intereſt ſhall* 264 li. 16 s. 5 d. *yield in* 15 *moneths at the ſame rate?*

Set down your numbers in Decimals, as in the Example following appeareth, ſo ſhall you find the Decimal of 264 *li.* 16 *s.* 5 *d.* to be 264.8208, all the reſt being whole numbers, having no fractions joyned with them we neglect, and work with them as they are, ſo will the ſeveral numbers of your queſtion (if rightly diſpoſed) ſtand as followeth.

li.	*mo.*	*li.*	*li.*	*mon.*
100	12	6	264.8208	15
	100		6	
	1200		1588.9248	
			15	
			79446240	
			15889248	
	1200)		23833.8720	(19.8615

1200 or
11833 *li.* *s.* *d.* *q.*
19 17 2 3
10800 *fere*
10338

9600
7387

7200
1872

1200
6720

6000
720

Your numbers being thus orderly diſpoſed, you muſt according to the Rule before delivered in the firſt

first Part, multiply the first and second terms together, which in this Example are 100 and 12, whose product is 1200, which is your Divisor; Then multiply the three last terms one into another, as 264,8208 (which is the Decimal of 264 *li.* 16 *s.* 5 *d.*) by 6, and the product thereof will be 1588.9248, which number again multiplyed by 15, (which is the last term) the product will be 23833.3720 which is your Dividend, and this number being divided by your former product, giveth in the quotient 19.8615, which is the Decimal of 19 *li.* 17 *s.* 2 *d.* 3 *q.* *fere*, and so much doth the simple interest of 264 *li.* 16 *s.* and 5 *d.* amount unto in 15 moneths, after the rate of six *per centum* for a year.

Question 2.

If the carriage of 23 *hundred and* 3 *quarters of any thing* 127 *miles, cost* 4 li. 13 s. 6 d. *what shall the carriage of* 47 *hundred and an half of such like commodity cost, being carried* 381 *miles.*

Place your numbers in order as in the following Example doth appear, then multiply the first and second terms together for your Divisor, and the three last one into another for your Dividend, and so will the quotient of this division answer the question demanded, and the work will stand as followeth.

```
C.      miles     li.      C.      miles
23.75 — 127 — 4.675 — 47.50 — 381
127               47.50
-----             ------
16625             23375
4750              32725
2375              18700
-----             ------
3016.25           222.06250
                  381
                  ---------
                  22206250
                  177650000
                  66618750
                  ---------
3016.25)84605.81250 (28.050
                  ....
                  ---------
                  603250
                  2426081
                  ---------
                  2413000
                  15081250
                  ---------
                  15081250
                  00000000
```

Here you see that the first and second terms multiplied together produced 3016.25, which must be your Divisor, and the three last terms being multiplyed one into another, produce 84605.81250, which number divided by 3016.25, giveth in the quotient 28.050, which Decimal representeth 28 *l.* one shilling, and so much will the carriage of 47 hundred

hundred and a half cost, being carried 381 miles?

Thus have I shewed the use of decimal Arithmetick in such questions as concern the *Golden Rule*, or *Rule of Three*; both Direct, Reverse, and Compounded, by an Example or two in each rule, and those compounded of fractions sufficient, I should now proceed to questions in *Fellowship*, with and without Time, as also *Barter*, *Alligation*, the Extraction of the Square and Cube Roots, &c. but forasmuch as these last mentioned Rules depend only upon the *Rule of Three*, as by Examples in the first part doth plainly appear, I shall therefore save that labour, and spare my Reader the pains of practising questions which wholly depend upon that which I (by this time) suppose him perfect in; Yet if the Reader be desirous to make trial of any such question for his own satisfaction, he may either make trial of those questions in the former part of this Book in those several Rules, reducing the numbers there given into Decimals, or if he please, he may frame questions according to his own fancy. And thus I shall conclude this second Part.

The End of the Second Part.

An Appendix,

To the Second Part:

Section 1.

Of Exchange of the Coins, Weights, and Measures of one Country, with the Coins, Weights and Measures of another Country.

TO perform this work, there is nothing required more then the *Golden Rule*, if first the *Rate* or *Proportion*, between the *Coins*, *Weights*, and *Measures* of any two *Countries* be first known which is best obteined by experience, rather then taken upon trust, all that in this place I shall do, is only to instruct the ingenious in the manner of Work, and make use of such Rates or Proportions as I find set down by Mr. *Lewis Roberts* Merchant, in his *Map of Commerce*.

Question 1.

How many Riders (each Rider containing 1 l. 1 s. 2 d. 2 q. *Sterling) shall I receive for* 251 l. 6 s. 4 d. 2 q. *Sterling?*

Facit

Facit 237 Riders.

	l.	*s.*	*d.*	*Rider*		*li.*	*s.*	*d.*	
If	1	1	$2\frac{1}{2}$ *give*	1	*how many Riders shall*	251	6	$4\frac{1}{2}$	*give?*

Here if you reduce your numbers to their least Denominations, or set them down in Decimals, and multiply and divide according to the Golden Rule, you shall find in your quotient 237, and so many *Riders* ought to be received for 251 *l.* 6 *s.* 4 *d.* 2 *q.* Sterling.

Question 2.

How many French-Crowns *(each French Crown being valued at* 6 s. *Sterling) shall I receive for* 492 l. 18 s. *Sterling?*

Facit 1643 French Crowns.

	s.		*F.C.*		*li.*	*s.*	
If	6	*give*	13	*what shall*	492	18	*give*

Multiply and divide according to the Golden Rule, and you shall have in your quotient 1643, and so many French Crowns are to be received for 492 *l.* 18 *s.* Sterling.

Question 3.

A Merchant delivered at Paris 1643 *Crowns of* 6 *s. Sterling the piece, how many pounds Sterling ought to be received at* London?

Answer 492 *l.* 18 *s.* *Sterling.*

Crown	*s.*	*Crowns*	
If 1	*give* 6,	*what shall* 1643	*give?*

Multiply and divide, and you shall have in your quotient 492 *l.* 18 *s.* and so much Sterling Money ought to be delivered at *London*, for 1643 French Crowns, of 6 *s.* the Crown Sterling.

Question 4.

If 3 *yards at* London, *be* 4 *Ells at* Antwerp, *how many yards at* London *make* 84 *Ells at* Antwerp?

Ells Antwerp	*yards* London	*Ells Ant.*
4	3	84

Facit 63.

And so many yards at *London*, are equal to 84 Ells at *Antwerp*.

Question 5.

How many yards of London *make* 27 *Ells of* Antwerp, *when* 100 *Ells of* Antwerpe *make* 60 *Ells of* Lions, *&* 20 *Ells of* Lions *make* 25 *yards of* London?

The

The first Work.

Ells Lions	*yards* London	*Ells* Lions
20	25	60
	60	
	1500	
	75	*facit* 75

That is 75 yards of *London* is equal to 100 Ells of *Antwerp.*

The second Work.

Ells Antwerp,	*yards* Lon.	*Ells* Antwerp.
100	75	27
	27	
	525	
	150	
		yards of London
	2025	*facit* 20. 25

Question 6.

If 100 li. *Sterling be* 104 li. 6 s. 4 d. *Flemish, what is one pound Sterling worth ?*

li. Sterl.

li. Sterl.	li.	s.	d.	li. Sterl.
100	134	6	4	1

20

2686
12

5372
26864

32236

Facit 322 *pence* $\frac{36}{100}$ of a peny.

Question 7.

How many Ells of Franckford *make* $42\frac{1}{4}$ *Ells of* Vienna *in* Auſtria, *when* 35 *Ells of* Vienna *make* 24 *at* Lions; 3 *Ells of* Lions, 5 *Ells of* Antwerp, *and* 100 *Ells of* Antwerp, 125 *Ells at* Franckford.

First-Work.

Ells Ant.	*Ells* Franck.	*Ells* Ant.
100	125	5

5

625

Facit 6.25 or 6 Ells and a quarter of *Franckford* equal to 3 Ells of *Lions*.

Second

Second Work.

Ells Lions,	*Ells* Franck.	*Ell* Lions.
3	6.25	24

6.25
24
―――
2500
1250
―――
3)150.00(50
15
0

Facit 50 Ells of *Franckford*, equal to 35 Ells of *Vienna*.

Third Work.

Ells Vien.	*Ells* Franck.	*Ells* Vien.
35	50	42.2

35) 2112.50 (60.35   2112.50

210
125
―――
105
200
―――
175
25

Facit

Facit 60. 35. or 61¼ Ells of *Franckford*, equal to 35 Ells of *Vienna*.

Thus have I given you a few Examples of Exchanges, I will now insert some few Tables derived from Mr. *Lewis Roberts* his Map of Commerce afore-said, of the truth of which, I am not a competent Judge, but shall leave that to the scrutiny of such as have occasion to trade into Forreign Countries.

A Table shewing what one pound of *Avoirdupois* weight at *London* maketh in divers eminent Cities, and other remarkable places.

One pound of Avoirdupois *weight at* London *makes at*

	lb.
Antwerpe	.9615
Amsterdam	.9
Abbeville	.91
Ancona	1.282
Avignon	1.12
Burdeaux	.91
Burgoyne	.91
Bolonia	1.25
Bridges	.98
Calabria	1.3698
Calais	1.07
Constantinople	.8474 Loder;
Deep	.91
Dantsick	1.16
Ferrara	1.3333
Florence	1.282
Flanders in general	1.06
Geneva	.9345

Genoa

One pound of Avoirdupois weight at London, makes at

	lb.
Genoa	1.4084 suttle
	1.4285 grosse
Hamburg	.92
Holland	.95
Lixborne	.881
Lions	1.07 common weight
	.98 silk weight
	.9 customers weight
Legorn	1.3333
Millan	1.4285
Mirandola	1.3333
Norimberg	.88
Naples	1.4084
Paris	.89
Prague	.83
Placentia	1.3888
Rochell	1.12
Rome	1.27
Rovan	.875 by Vicont
	.9017 common weight
Sivill	1.08
Tholousa	1.12
Turin	1.2195
Venetia	1.5625 suttle
	9433 grosse
Vienna	.813

The

The use of the preceding Table.

How much weight at Bolonia, *will* 655 l. *Avoirdupois make?*

Look in the Table for *Bolonia*, and right against it you shall find 1.25, which sheweth that one pound Avoirdupois at *London* is equal to 1.25 *l.* at *Bolonia*, Therefore say by the Rule of Three:

If 1 *l.* Avoirdupois give 1.25 *l.* at *Bolonia*, what shall 655 *li.* Avoirdupois give? *Answer* 818.-75. As by the operation following doth appear.

li.	*li.*	*li.*
1	125	655
	655	
	625	
	625	
	750	
	81875 or 818$\frac{3}{4}$.	

A

A Table shewing what one pound Weight in divers forreign Cities, and remarkable Places, maketh at London of *Avoirdupois* Weight.

One pound weight in		*makes at London of Avoirdupois weight.* lb.
ANtwerpe		1.04
Amsterdam		1.1111
Abbeville		1.0989
Ancona		.78
Avignon		8928
Burdeaux		1.0989
Burgoyne		1.0980
Bolonia		.8
Bridges		1.0204
Calabria		.73
Calais		.9345
Deepe		1.0989
Dantsick		.862
Ferrara		.75
Florence		.78
Flanders in general		.9433
Geneva		1.07
Genova	subtle	.71
Genova	grosse	.7

Ham-

One pound weight in … *makes at London of* Avoirdupois *weight*

One pound weight in		lb.
Hamburg		1.0865
Holland		1.0526
Lisbone		1.135
Lions	common weight	.9345
	silk weight	1.0204
	custom weight	1.1111
Legorne		.75
Millan		.7
Mirandola		.75
Norimberg		1.136
Naples		.71
Paris		1.1235
Prague		1.2048
Placentia		.72
Rochel		.8928
Rome		.7874
Rovan	by Vicount,	1.1428
	common weight,	1.1089
Sivil		.9259
Tholousa		.8928
Turin		.82
Venetia	suttle,	.64
	grosse:	1.06
Vienna		1.23

1716
V

The

The use of the foregoing Table.

In 7652 *li. weight at* Mirandola, *how many pound Weight of* Avoirdupois.

Look in the Table for *Mirandola*, and right against it you ſhall find .75, which ſheweth that one pound *Avoirdupois* is equal to the 75 or $\frac{3}{4}$ of a pound at *Mirandola*, wherefore ſay by the Rule of Three.

If 1 *l.* at *Mirandola*, gives .75 or $\frac{3}{4}$ of a pound *Avoirdupois*, what ſhall 7652 *l.* of *Mirandola* give? *Anſwer* 5739, as by the operation doth appear.

li.		
1	.75	7652
		75
		38260
		53564
		573900

A Table

A Table reducing English Ells to the Measures of divers forreign Cities and remarkable places.

One Ell at London *makes at*

Place		Measure	
Amsterdam		1.6949	Ells
Antwerpe		1.6666	Ells
Bridges		1.64	Ells
Arras		1.65	Ells
Norimberg		1.74	Ells
Colen		2.08	Ells
Lisle		1.66	Ells
Mastricht		1.57	Ells
Frankford		2.0866	Ells
Dantsick		1.3833	Ells
Vienna		1.45	Ells
Paris		.95	Aulns
Rovan		1.03	Aulns
Lions		1.0166	Aulns
Calais		1.57	Aulns
Venice	linnen	1.8	Braces
Venice	silke	1.96	Braces
Lucques		2.	Braces
Florence		2.04	Braces
Milan		2.3	Braces
Legorn		2.	Braces
Madera Isles		1.0328	Braces

One Ell at Lond. *make, at*	Sivill	1.35	
	Lisbone	1.	Vares
	Castilia	1.3875	Vares
	Andaluzia	1.4625	Vares
	Granado	1.3625	Vares
	Genoa	4.8083	Palmes
	Saragosa	.55	Canes
	Rome	.56	Canes
	Barselona	.7225	Canes
	Valentia	1.2125	Canes

The use of this Table.

In 632 Ells at London, *how many Braces at* Florence ?

Look in the Table for *Florence*, and right against it you shall find 2.04, which sheweth that one Ell at *London*, maketh at *Florence* 2.204 Braces, wherefore say by the Rule of Three.

If one Ell at London *give 2.04 Braces at* Florence, *how many Braces shall 632 Ells give ?* Answer 1289.28, as by the operation following doth appear.

$$1 \text{———} 2.04 \text{———} 632$$

$$\begin{array}{r} 632 \\ \hline 408 \\ 612 \\ 1224 \\ \hline 1289|28 \end{array}$$

A Table reducing the Measures of divers forreign Cities and remarkable places; to *English* Ells.

One Ell at	AMsterdam	*makes at London*	.59	*Ells*
	Antwerpe		.6	
	Bridges		.6097	
	Arras		.606	
	Norimberg		.5474	
	Colen		.4807	
	Lisle		.6024	
	Mastricht		.6369	
	Frankford		.4792	
	Dantsick		.7228	
	Vienna		.9896	
One Auln at	Paris		1.0526	
	Rovan		.9708	
	Lions		.9836	
	Calais		.6369	
One Brace at	Venice linnen		.5555	
	Venice silke		.5102	
	Lucques		.5	
	Florence		.4901	
	Millan		.4347	
	Legorn		.5	
	Madera Isles		.9681	

Sivil

One Vare at	Sivil	*makes at* London	.7409
	Lisbone		1.
	Castilia		.7207
	Andalusia		.7339
	Granado		.7339
One Palm at Genoa			.2079
One Cane at	Saragosa		1.8181
	Rome		1.7857
	Barselona		1.4035
	Valentia		.8247

The use of this Table.

In 5727 *Braces at* Legorn, *how many Ells* English.

Look in the Table for *Legorn*, and right against it you shall find .5, which sheweth that one Brace at *Legorn* maketh at *London* .5 or half an Ell, wherefore say by the Rule of Three.

If one Brace at Legorn *give* .5 *Ells at* London, *what shall* 5727 *Braces give?* Answer 2863.5, as by the work appeareth.

```
1 ———— .5 ———— 5727
                 .5
              ———————
              2863.5
```

Section 2.

Concerning Interest and Annuities.

The first Table shewing what one pound being forborn any number of years under 31, will amount unto, accounting interest upon interest, after the rate of 6 per cent.

Y.	6 per cent.	Y.	6 per cent.	Y.	6 per cent.
1	1,06	11	1,89829	21	3,89956
2	1,1236	12	2,01219	22	3,60353
3	1,19101	13	2,13292	23	4,81975
4	1,26247	14	2,26090	24	4,04893
5	1,33822	15	2,39655	25	4,29187
6	1,41851	16	2,54035	26	4,54938
7	1,50363	17	2,69277	27	4,82234
8	1,59384	18	2,85433	28	5,11168
9	1,68947	19	3,02559	29	5,41838
10	1,79084	20	3,20713	0	5,74349

The

The firſt Column of this Table having *Y* at the top thereof, beginning at 1, and ſo proceeding to 30, ſignifie years, and the number in the next Column anſwering thereunto do ſhew what one pound is worth, being forborn any number of years under 31, which Table is made according to this proportion.

As 100 to 106, ſo is 1 to 1.06

and again,

As 100——106——106——1.1236

and thirdly.

As 100—106——1.1236——1.19101

Et ſic ad infinitum.

The uſe of this Table.

What 136 l. 15 s. 6 d. *will amount unto, being forborn* 20 *years, after the rate of* 6 per centum, *intereſt upon intereſt.*

Look in the Table for 20 years, and right againſt in the broader Column, you ſhall find 3.20713, which ſhews that one pound being forborn 20 years will be augmented to 3.20713. Then if you reduce your 136 *li.* 15 *s.* 6 *d.* into a Decimal, either by the Tables in the Second Part, or by the Scales in the Third Part of this Book, you ſhall find it to be 136.775. Wherefore ſay by the Rule of Three Direct.

If

If one pound being forborn 20 *years will be augmented to* 3.20713, *to how much will* 136.775 li. *be augmented to in the same time.* Answer, *to* 438 *li.* 13 *s.* 1 *d.* 1 *q.* as by the operation following doth appear.

li.	*li.*	*li.*
1	3.20713	136.775

```
            3.20713
            136.775
           --------
           1603565
          2244991
         2244991
        1924278
        962139
       320713
      ------------
       438.65520575
```

Or

438 *li.* 13 *s.* 1 *d.* 1 *q.*

The

The Second Table, sheweth what one pound will amount unto, being forborn any number of years under 31, at 6 per cent. interest upon interest, the Annuity being to be paid yearly.

Y.	6 per cent.	Y.	6 per cent.	Y.	6 per cent.
1	1,00000	11	14,97164	21	39,99272
2	2,06000	12	16,86994	22	43,39228
3	3,18360	13	18,88213	23	46,99582
4	4,37461	14	21,01506	24	50,81557
5	5,63709	15	23,27596	25	54,86451
6	6,97531	16	25,67252	26	59,15638
7	8,39383	17	28,21287	27	63,70576
8	9,89746	18	30,90565	28	68,52810
9	11,49131	19	33,75999	29	73,63979
10	13,18079	20	36,78559	30	79,05818

The use of this Table.

What will an Annuity of 20 li. *payable yearly, be aug-*

augmented unto in 12 years, being all that time forborn, accounting interest upon interest at 6 per cent. per annum.

Look in the first column of the Table for 12 years, and right against it in the next column you shall find 16,86994, which shews that 1 *li.* Annuity payable yearly, being forborn 12 years, will amount unto 16.86994, wherefore say by the Rule of Three Direct.

If 1 *pound Annuity forborn* 12 *years give* 16.86994, *what shall an Annuity of* 20 *pound a year give, being forborn the same term of* 12 *years?* Answer 337 *li.* 7 *s.* 11 *d.* 3 *q. fere,* as in the operation doth appear.

16.86994 —— 20
20
———
337.39880
or
337 *l.* 7 *s.* 11 *d.* 3 *q. fere*

The

The Third Table sheweth what one pound being forborn any number of years under 31 is worth in ready money, rebating yearly, after the rate of 6 per cent. *interest upon interest.*

Y.	*6 per cent.*	*Y.*	*6 per cent.*	*Y.*	*6 per cent.*
1	,943396	11	,526787	21	,294155
2	,809996	12	,496989	22	,277505
3	,839619	13	,468839	23	,261797
4	,792093	14	,442300	24	,246978
5	,747258	15	,417263	25	,232998
6	,704960	16	,393646	26	,219810
7	,665057	17	,370364	27	,207367
8	,627412	18	,351343	28	,195630
9	,591898	19	,330512	29	,184536
10	,558394	20	,311804	30	,174110

The making of the Table.

As—106—100—.943396—889996

and

and again,

As 106——100——.889996——839619

Et ſic ad infinitum.

If 356 li. *be payable at the end of* 7 *years, what is it worth in ready money, diſcounting or rebating after the rate of* 6 per cent. *intereſt upon intereſt.*

Look in the Table for 7 years, and againſt it you ſhall find .665057, being the ready money which 1 *li.* is worth payable at 7 years end, wherefore ſay by the Rule of Three.

If 1 li. *in* 7 *years rebate or decreaſe to* .665057, *to what will* 356 li. *rebate or decreaſe in the ſame time?* Anſwer, *to* 170 *li.* 5 *s.* 1 *d.* as by the operation doth appear.

1————.665057————
256

3990342
3325285
1330114

170.254592
or
170. *l.* 5 *s.* 1 *d.*

The

The Fourth Table sheweth the present worth of one pound Annuity, to continue any number of years under 31, and payable yearly after the rate of 6 per cent. *interest upon interest.*

Y.	*6 per cent.*	*Y.*	*6 per cent.*	*Y.*	*6 per cent.*
1	0,94339	11	7,88687	21	11,76305
2	1,83339	12	8,38384	22	12,04158
3	2,67301	13	8,85268	23	12,30337
4	3,46510	14	9,29498	24	12,55035
5	4,21236	15	9,71224	25	12,78335
6	4,91732	16	10,10589	26	13,00316
7	5,58238	17	10,47725	27	13,01053
8	6,20979	18	10,82760	28	13,40616
9	6,80169	19	11,15811	29	13,59071
10	7,36008	20	11,46902	30	13,76482

The use of this Table.

What is the present Rent or Annuity of 25 *pound*
per

per annum *worth payable yearly, for* 22 *years, accounting Interest upon Interest at* 6 per centum.

Look in the Table for 21 years, and right against it you shall find 11.76407, which is the present worth of one pound Annuity for 21 years, wherefore say by the Rule of Three.

If an Annuity of 1 li. per annum *for* 21 *years be worth* 11.76407 *ready money, what is an Annuity of* 25 li. per annum *worth in ready money for the same time?* Answer 294 *li.* 2 *s.* 0 *d.* 1 *q.* as by the operation following doth appear.

1 —— 11.76407 —— 25

25

5882035

2352814

294.1075

or

294 *l.* 2 *s.* 0 *d.* 1 *q.*

The

The Fifth Table sheweth what Annuity payable yearly, one pound will purchase for any number of years under 31, after the rate of 6 per cent. compound interest.

Y.	6 per cent.	Y.	6 per cent.	Y.	6 per cent.
1	1.06000	11	.12679	21	.08500
2	.54363	12	.11926	22	08304
3	.37411	13	.10297	23	.08127
4	.28859	14	.10758	24	.07967
5	.23739	15	.10296	25	.07822
6	.20336	16	.09895	26	.07690
7	.17913	17	.09544	27	.07569
8	.16103	18	.09235	28	.07459
9	.14702	19	.08962	29	.07357
10	.13586	20	.08718	30	.07264

The use of this Table.

What Annuity to begin presently, and to continue 28 years, payable at yearly payments, will 640 li. *purchase, accounting compound Interest after the rate of* 6 per cent.

Look in your Table for 28 years, and right against it in the next column you shall find .07459, which shews that one pound ready money will purchase an Annuity worth .07459, and to continue 28 years, wherefore say by the Rule of Three.

If one pound ready money will purchase an Annuity worth 07459 *to continue* 28 *years, what Annuity shall I purchase for the same time, paying* 640 li. *ready money?* Answer 47 li. 14 s. 9 d. *as by the operation doth appear.*

1 ——— .07459 ——— 640

640

———

29836

44754

———

47.73760

or

47 *l.* 14 *s.* 9 *d.*

FINIS.

INSTRUMENTAL ARITHMETICK.

The Third Part.

TEACHING,

By a new Artifice, (not heretofore Published, to my Knowledge, in any Language) The manner how to set down any *Decimal Fraction* required: Or a *Decimal Fraction* being given, to find the value thereof in *English Money*, *Weight* or *Measure*; by inspection only. By certain SCALES contrived, suitable to the *Coyns*, *Weights* and *Measures* now used in *England*.

By *Will. Leybourn*.

LONDON,

Printed Anno Dom

INSTRUMENTAL ARITHMETICK.

The Third Part.

THe *Arithmetick*, of which we now come to treat, and which I call *Instrumental Arithmetick*, is not any new kind of *Arithmetick*, but is indeed the same with *Decimal Arithmetick* before taught; onely, whereas in *Decimal Arithmetick* there were certain *Tables* made of *Money*, *Weight* and *Measure*, by help of which the Decimal of any Fraction of *Money*, *Weight* and *Measure*, might be set down (as it were) in whole numbers, here in this *Instrumental part*, we have contrived certain *Scales* of *Money*, *Weight* and *Measure*, equally divided into the several *Denominations* into which the *Weights* and *Measures* for which they are contrived, may be equally divided. Unto all which Scales there is joyned a Scale of 100, 1000 or 10000, *equal parts*, according to the length of the Scale, so that by *inspection* only you may readi-

ly and exactly without *addition* (as in using the forementioned *Tables* you must necessarily do) set down the *Decimal Fraction* of any part of *Money*, *Weight* or *Measure*, with great celerity and exactness, if the *Scale* be any thing well divided, and be but of a reasonable length.

Now the Scales which I have chiefly made choice of in this Work; as being of most use with English men; (though other Scales may be made for the *Coyns*, *Weights* or *Measures* of any other *Countrey* as well, and upon the same ground) are chiefly these, *viz.*

1 Of Money.
2 Of Troy Weight.
3 Of Avoirdupois great Weight.
4 Of Avoirdupois little Weight.
5 Of Liquid measures.
6 Of dry measures.
7 Of Long measures.
8 Of Dozens.

Unto every of these Scales, is joyned another Scale of 100 or 1000 equal parts, these Scales are made to face one another, so that if you look upon any one Division in the one, you shall also discern plainly what Division or part of a Division answereth thereunto in the other.

These Scales being thus disposed, as they may easily be upon any *Ruler* of *Silver*, *Brass* or *Wood*; but best of all upon a *square Ruler*, made in form of a *Parallelepipedon*, will by inspection only give you any Decimal fraction required without *Addition*, or (on the contrary) reduce the fraction into the known parts

parts of the Integer, by inſpection alſo, without *Subſtraction.*

Let thus much ſuffice for a general deſcription of what I mean by Scales, the particular deſcription of them will more plainly appear, when we treat of Numeration upon the Scales, unto which we ſhall now proceed. But firſt take a view of the Scales as they are here diſpoſed, and as they may be ſet upon ſuch a Ruler as I have here mentioned.

Numeration upon the Scales.

THe Scales here to be deſcribed are in number eight, as hath been already ſhewed, and as by the figure of them appear. *Now* Numeration *upon a* Scale, *is to find upon what part of the* Scale, *any* number *upon the ſame* Scale *will fall.*

We will begin with the firſt, and ſo proceed till we have given an Example in every one.

1 The firſt Scale is of *Engliſh Money*, and is divided into 24 equal parts, which repreſent 24 pence or 2 ſhillings, theſe parts are numbred with Arithmetical figures, from the beginning thereof, by 1, 2, 3, 4, 5, &c. to 24, each diviſion repreſenting one peny, and the whole 24 diviſions repreſenteth 24 pence or 2 ſhillings; So that where the figure 1 ſtandeth, that part of the Scale repreſenteth one peny, where the figure 2 ſtandeth, it repreſenteth two pence, where the figure 18 ſtand-

eth, it representeth 18 pence, or one shilling six pence, and so of any other figure of the same Scale.

Then because there are four farthings contained in a peny, each of these pence (or divisions) is sub-divided into 4 other equal parts by short lines, every one of these representing one farthing, so is the whole Scale divided in all in 96 equal parts, which are the number of farthings contained in two shillings. Thus if you look into the Scale of Money for 8 pence 3 farthings, you shall find it at the letter *a*, which letter is here put onely for examples sake, Also if you would find in the Scale the place of 18 *d.* half peny, you shall find it at *c*, and thus may you find the place of any number of pence and farthings under two shillings upon the Scale.

Unto this Scale of money (as to all the rest of the Scales) there is joyned another Scale which I shall alwayes hereafter call the Scale of 1000, the use of which Scale is this. When you have found any Number of pence or farthings upon the Scale of Money, you shall find upon the Scale of 1000, what parts of a thousand is the Decimal of those pence and farthings: Thus when in the Scale of money you find at the letter *a* 8 pence 3 farthings, if you cast your eye directly cross to the Scale of 1000, you shall find 364 to stand directly against 8 *d.* 3 *q.* which 364 is the Decimal of 8 *d.* 3 *q.* Also, if you find upon the Scale of money 18 *d.* half peny, which is at the letter *c*, you shall find against it in the Scale of 1000, this number .771, which is the Decimal of 18 *d.* half peny. And in this manner may the Decimal of any number of pence or farthings under two shillings be most easily and exactly obtained.

Now

Now on the contrary, suppose a Decimal Fraction were given, representing some part of *English Coin*, if you look in the Scale of 1000 for your Number given, right against it in the *Scale of Money* you shall find what number of pence and farthings is represented thereby. As for Example, Suppose .364 were a Decimal given, and it were required to find what part of Coin it doth represent. Look in the Scale of 1000 for the Number 364, and right against it you shall find 8 pence 3 farthings. Also if .771 were a Decimal given, if you look in the Scale of 1000 for .771, you shall find against that number 18 pence 2 farthings. And thus of any other.

By what hath been already said, it may be easily discerned of what exceeding expedition these Scales thus disposed are of, for I dare affirm, that I will set down 2 (if not 3) numbers, by the Scale, as soon as one by the Tables, and if the Scale be but of any reasonable length, altogether with as much exactness, but if I should vary an unite in the last place, in my estimation in the Scale of 1000, it is not any thing material.

I have been very tedious in shewing the use of these Scales to find the fraction parts of *Money*; but the reason is, because I intend to be the briefer in the rest, for *weight* and *measure*, the manner of working (when the division of the Scale is known) being the same in all respects without the least alteration.

2 The second Scale is of *Troy weight*, two peny weight being the Integer, which Scale is divided into 48 equal parts or divisions, each of which divisions

visions contains one grain, and are numbers by Arithmetical figures at every three grains by 3, 6, 9, 12, &c. to 24, and at the place where 24 should stand, there standeth P W, which signifieth one peny weight, or 24 grains, this P W standeth in the middle of the line. Then is the same Scale continued farther by Arithmetical figures, 3, 6, 9, 12, &c. as before to 24, and there is written P W again, representing two peny weight, or 48 grains.

The Scale being thus divided, it is easie to find the place where any number of Grains under 48 shall be upon the Scale; As for example, if it be required to find where 8 peny weight shall fall, look upon the Scale of Troy weight, from the beginning thereof, and count the figures 3 and 6, then count also two of the smaller Divisions, and that makes 8 grains, which you shall find to stand at the letter *d*, which is the place of 8 grains; Also if upon the Scale you would find the place of one peny weight 10 grains, you shall find it at the letter *e*, and so of any other number of grains under 48, or two peny weight.

But if you had a Decimal given, and would know what number of grains it representeth, if you seek your Decimal given in the Scale of 1000, right against it in the Scale of Troy weight, you shall find the number of grains represented thereby.

Example. Let .167 be a Decimal fraction given. If you look in the Scale of 1000, for .167, right against it in the scale of *Troy weight*, you shall find 8 peny weight.

Also if .708 were a Decimal given, if you seek 708 in the scale of 1000, right against it you shall fine 1 peny weight 10 grains.

3 The third scale is of *Avoirdupois great weight*, 28 pounds, or one quarter of an hundred, being the Integer, this scale is numbred by 1, 2, 3, 4, &c. to 28, which 28 representeth 28 *l.* or a quarter of a hundred, and each of those is subdivided into four small parts, each representing one quarter of a pound.

Now if you would know what is the Decimal of any number of pounds or quarters under 28, if you seek the number of pounds in the scale of *Avoirdupois great weight*, right against it in the scale of 1000, you shall find the decimal thereof.

Thus if it were required to find the Decimal of 8 pound and an half, if you lock upon the Scale for 8 pound and an half, you shall find it at the letter *g*, and right against it in the Scale of 1000 you shall find .304, which is the Decimal of 8 pounds and an half.

On the contrary, suppose .304 were a Decimal given, and it were required to find what part of *Avoirdupois great weight* were represented thereby, if you look in the Scale of 1000 for 3.04, right against it in the Scale of *Avoirdupois great weight*, you shall find 8 pound and an half.

4 The fourth Scale is of *Avoirdupois little weight*, 16 ounces or one pound being the Integer; This Scale is first divided into 16 equal parts, and numbred by 1, 2, 3, 4, &c. to 16, each Division representing one Ounce. Then again, each of these ounces is sub-divided into 8 other smaller parts or divisions, each of which divisions representeth two Drams; but if your Scale be large enough, you may have each ounce divided into 16 equal parts or divisions, each division representing one Dram.

Now

Now to find the Decimal belonging to any number of Ounces and Drams, repair to the Scale of *Avoirdupois little weight*, and on it find the quantity of ounces and drams required, and right against it in the Scale of 1000, you shall have the Decimal thereof.

Thus if it were required to find the Decimal of *6* ounces and *6* drams, if you look this in the Scale of *Avoirdupois little weight*, you shall find it at the letter *b*, and right against it in the Scale of 1000, you shall find 398, which is the Decimal of 6 ounces and 6 drams.

But if .398 were a Decimal given, and it were required to find the value thereof in *Avoirdupois little weight*, if you look for .398 in the Scale of 1000, right against it in the other scale you shall find *6* ounces *6* drams.

5 The Fifth Scale is of *Dry Measures*, one *Quarter* or 8 *Bushels* being the Integer; This Scale is first divided into 8 equal parts, and numbered by 1, 2, 3, &c. to 8, each of which divisions representeth a Bushel, and each of those parts is again sub-divided, first into 4 equal parts or divisions each representing one *peck*, and then those again sub-divided into 4 other smaller parts, representing *quarters*, *halves*, and *three quarters* of a peck.

Now if you would know the Decimal belonging to any number of Bushels (under 8 Bushels or one quarter) Pecks and parts of a Peck, if you seek the number of Bushels, Pecks, and parts of a Peck in the Scale of *Dry Measures*, right against it in the Scale of 1000, you shall have the Decimal required.

As

As for Example, if it were required to find the Decimal belonging to 5 Bushels 2 Pecks, and half a Peck, if you look into the Scale of dry Measures you shall find 5 bushels, 2 pecks and an half to stand at the letter *k*, and right against it in the Scale of 1000, you shall find .702, which is the decimal answering to 5 bushels, 2 pecks and an half.

But if .702 were a decimal given, and it were required to find what number of bushels, pecks, and parts of a peck it representeth, if you look in the scale of 1000 for .702, you shall find against it in the scale of *Dry measures*, 5 bushels, 2 pecks and an half.

6 The sixth Scale is of *Liquid Measures*, the Integer being 36 Gallons or one Barrel, this Scale is divided first into 36 equal parts or divisions, and numbred by 5, 10, 15, &c. to 36, then every of these divisions is again subdivided into 4 other small divisions, each representing a quart, but (if the Scale be large enough) you may subdivide each Gallon into 8 parts, so will every part represent one pint.

Now to find the decimal belonging to any number of *Gallons* (under 36 Gallons or one Barrel) quarts or pints, repair to the Scale of *Liquid measures*, and seek there upon the Scale, the number of gallons, quarts or pints, and against it in the Scale of 1000, you shall find the decimal thereunto belonging.

So if it were required to find a decimal representing 10 gallons and two quarts, or 4 pints, which is all one, if you seek in the scale of Liquid measures for 10 gallons, 2 quarts, you shall find it at the letter *m*, against which in the scale of 1000, you

shall

shall find .292, which is the decimal of 10 gallons 4 pints.

Likewise if .292 were a decimal given, and it were required to find what number of gallons, quarts or pints were represented thereby if you look in the scale of 1000 for .292, right against it in the scale of Liquid measures, you shall find 10 gallons, 2 quarts, or 4 pints.

7 The seventh Scale is of *Long Measures*, the Integer being *Yards* or *Ells*, this scale is divided into 4 equal parts, and numbred by 1, 2, 3, 4, representing 1 quarter, 2 quarters, 3 quarters or 4 quarters of a Yard or Ell, these are again sub-divided first into 4 other equal parts, representing *Nailes*, and those may be again sub-divided at pleasure if need be.

Now if you would know what decimal belongeth to any number of *quarters or nailes of a yard or Ell*, if you seek the number of quarters and nailes in the scale of *Long measure*, the scale of 1000 will give you the decimal thereof.

Thus if it be required to find the decimal belonging to 1 quarter and 3 nailes, if you seek this in the scale of *Long measure*, you shall find it to stand at the letter *o*, against which in the scale of 1000 you shall find .437, which is the decimal answering to 1 quarter and 3 nailes of a *Yard* or *Ell*.

Also if .437 were a decimal given, and it were required to find what quantity of *yards* or *ells* were represented thereby, if you look in the scale of 1000 for .437, you shall in the scale of *Long measure* find against it one quarter and 3 nailes.

8 The eighth and last scale is of *representative*

inches,

inches, the whole ſcale being divided into 12 equal parts, and numbred by 1, 2, 3, &c. to 12, and thoſe parts are again ſubdivided into halves, quarters, and half quarters, as Carpenters-Rules are uſually divided.

Unto this Scale (as unto all the other) there is joyned a Scale of 1000, this Scale will readily diſcover what is the decimal belonging to any number of Inches, halves or quarters, and the uſe is the ſame with the Scales before mentioned.

Thus I have given you a brief deſcription of theſe *Scales*, and the uſes of them, and do now ſuppoſe my Reader to be perfectly acquainted with the way of numbring or counting upon them; Wherefore I intend onely to give you a queſtion or two in the moſt uſual Rules of Arithmetick, and ſo conclude; for Decimal Arithmetick being already ſufficiently explained, I ſhall not need to repeat the Rules (or the manner of working them) again, but give you one Example, by which the exactneſs and expedition of theſe Scales may the more evidently appear, for when we work by Scales, it is ſuppoſed that we do not uſe *Vulgar*, but *Decimal Arithmetick* and Addition, Subſtraction, Multiplication, Diviſion, the Rule of Three, and indeed, all the other Rules of Arithmetick, are to be performed, as is before taught, the Scale ſerving only to avoid Reduction.

Addition.

VVHat *Addition* is, and the manner of working of it hath been already taught, both in the firſt and ſecond Parts, we will now come to an Example, which let be in *Addition of Engliſh Coin*, and let the ſums to be added be 36 *l.* 8 *s.* 8 *d.* 29 *l.* 0 *s.* 2 *d.* 31 *l.* 16 *s.* 9 *d.* and 6 *l.* 2 *s.* 5 *d.*

Firſt, ſet down 36 *l.* 29 *l.* 31 *l.* and 6 *l.* one under another, in ſuch order as you ſee here in the margine, drawing a line by the ſide of them as you ſee here done, and alſo a line under them.

36	
29	
31	
6	
—	—

This done, ſeeing that your firſt number to be ſet down to 36 *l.* is 8 *s.* 8 *d.* you muſt for the 8 *s.* (becauſe two ſhillings, which we called a Decade, or the tenth part of a pound, is made the Integer, in the Scale of Money) ſet down 4, which is done by memory, and after it make a comma, Then your next number to be ſet by 29 *l.* being 0 *s.* 2 *d*, for the 0 *s.* ſet down a Cypher; Thirdly, for your number to be ſet by 31 *l.* being 16 *s.* 9 *d.* for the 16 *s.* ſet down 8 decades, with a comma after it, and laſtly, the number to be ſet by 6 *l.* being 2 *s.* 5 *d.* for the 2 *s.* I ſet down 1 decade with a comma after it, and then will your work ſtand, as here you ſee.

36	4,
29	0,
31	8,
6	1,
—	—

Then take your ſcale in hand, and ſeeing your firſt

firſt number of pence are 8 *d.* look in your ſcale of money for 8 *d.* and againſt it in the ſcale of 1000, you ſhall find 333, which ſet 36 *l.* 8 *s.* behind the comma, then your next number of pence being 2 *d.* look in your ſcale for 2 *d.* and againſt it in the ſcale of 1000, you ſhall find 083, which ſet to 29 *l.* 0 *s.* behind the comma. Then your third number of pence being 9 *d.* look in your ſcale for 9 *d.* and againſt it in the ſcale of 1000, you ſhall find 376, which ſet to 31 *l.* 16 *s.* and laſtly, your laſt number of pence being 5 *d.* look in your ſcale for 5 *d.* and againſt it you ſhall find 208, which ſet to 6 *l.* 2 *s.* and then will your whole work ſtand, as here you ſee.

36	4,333
29	0,083
31	8,376
6	1,208
—	—

Your ſums being thus ſet down, which is done with more facility then you can imagine, till you make trial and be ſomething perfect therein, you muſt then add all the numbers together, as in Addition of Decimals, and you ſhall find the ſum of them to be 10314,000, Now to know this in money, is as eaſie as it was to ſet ſeveral ſums down, for the figures 103, which ſtand behind the down right line, are 103 *l.* and the figure 4 which ſtands between the down right line and comma, are 4 decades or 8 *s.* and being the reſt to the right hand are all Cyphers, they ſignifie neither pence nor farthings, ſo is the total of this Addition 103 *l.* 8 *s.* 0 *d.*

l.	
36	4,333
29	0,803
31	8,376
6	1,208
—	—
103	4,000

That the manner of working may appear more plain, I will give you

another short Example as difficult as I can invent, which I performed by a Scale of Wood but of 8 inches long. Let the sums to be added together, be these following.

li.	s.	d.	q.
332	17	4	1
159	6	8	1
217	5	3	3
709	9	4	1

First set down your several sums of pounds one under another as before, and draw a line by the side of them, and another under them. So will they stand as here you see.

332
159
217

1. Your sums of pounds being thus orderly placed and lines drawn, repair to your Scale and seeing your first number of shillings, pence and farthings is 17 *s.* 4 *d.* 1 *q.* for your 17 *s.* set down 8 Decades which is 16 *s.* with a comma after it, then will there rest to be set down 1 *s.* 4 *d.* 1 *q.* or 16 *d.* 1 *q.* which if you seek in your scale of money, you shall find to stand against it in the scale of 1000 this number .677, which is the Decimal of 1 *s.* 4 *d.* 1 *q.*

2. Your second number of shillings, pence and farthings is 6 *s.* 8 *d.* 1 *q.* for your 6 *s.* set down 3 decades, which is 6 *s.* and then there will remain 8 *d.* 1 *q.* which if you seek in your scale of money, you shall find to stand against it in the scale of 1000 this number .344, which is the decimal of 8 *d.* 1 *q.*

3. Your third number of shillings, pence, and farthings,

farthings, is 5 *s.* 3 *d.* 3 *q.* for your 5 *s.* set down 2 decades, which is 4 *s.* with a comma after it, then will there rest to be set down 1 *s.* 3 *d.* 3 *q.* or 15 *d.* 3 *q.* which if you seek in your scale of money, you shall find to stand against it in the scale of 1000, this number .656, which is the Decimal of 15 *d.* 3 *q.* or 1 *s.* 3 *d.* 3 *q.* and the three sums to be added together will stand as here you see.

l.	
332	8,677
159	3,344
217	2,656
709	4,677

These sums being added together according to the rule for Addition of Decimals, you shall find the sum of them to be 70914,677, now to know what this is in money, take notice that the 709 which stands to the left hand of the down right line are 709 pounds, and the figure 4, which stands between the down right line and the comma, are 4 decades or 8 *s.* but (because the first figure next after the comma is above 5, *viz.* 6) you must add 1 *s.* to the 4 decades, making them 9 *s*; then will there remain 177, wherefore if you look in the scale of 1000 for 177, you shall find against it in the scale of Money 4 *d.* 1 *q.* So is the whole sum of this Addition 709 *l.* 9 *s.* 4 *d.* 1 *q.* as by the preceding work doth appear.

¶ Here note, that when you had set down your 709 *l.* 4 decades or 8 *s.* there remained beyond the comma 677, which if you had sought in your scale of 1000, you should have found against it in the scale of Money 15 *d.* 1 *q.* or 1 *s.* 3 *d.* 1 *q.* (which is all one) as before, for it appeareth plainly by the Scale that 500 in the line of 1000 is equal to one shilling.

I might proceed farther in giving you Examples in *Weight* and *Measure* answerable to the *Scales*, but that would only make the Reader spend his time to little purpose, for being before acquainted with *Decimal Arithmetick*, and (as by this time I suppose he is) with *Numeration* upon the *Scales*, he cannot be deficient in the applying of the other *Scales* of *Weight* and *Measure* to the same purpose for which they were contrived, I having so largely exemplified the use of the *Scale* of *Money*.

Substraction.

SUbstraction (as hath been before said) is the taking one or more smaller sums out of one greater I shall only give you an Example or two, as I have taken the numbers from a Scale.

Example.

Delivered to a Gold-smith of old Plate 297 *ounces* 13 *peny-weight*, 19 *grains.*

Received of the same Gold-smith first 165 *ounces*, 11 *peny-weight and* 7 *grains, and after that received of the same Gold-smith* 32 *ounces* 19 *peny-weight and* 23 *grains, what plate remains in the Gold-smiths hands?*

Take your Numbers out of the Scale of Troy weight, and set them down as here you see.

ounces

	ounces
Delivered	29716,896
Received first	1655,646
Received more	329,979
Received in all	19815,625
Rests in the Gold-smiths hands	9911,271

or

ounces	peny-w.	gr.
99	2	13

Then add the several weights of Plate received, together, and they make 19815,625, or 198 ounces 11 peny-weight, 6 grains, which if you substract from 29716,896, or 257 ounces, 13 peny-weight, 19 grains, which was the quantity of plate delivered there will remain 9911,271, or 99 ounces, 2 peny-weight, 13 grains, and so much plate is still in the *Gold-smiths* hand. And let thus much suffice for *Substraction.*

Now we should proceed to *Multiplication* and *Division*, but when the numbers are taken from the Scale and set down, the manner of working doth not at all differ from *Multiplication* and *Division* of *Decimals* before taught. But that we are now a treating of *Instrumental Arithmetick* (because I will not alter the method before used, in the first and second parts of this Book;) I will therefore here shew you how *Multiplication*, *Division* and the *Extraction* of the *Square* and *Cube Roots*, may be *Instrumentally* performed by *Rods*, commonly called *Nepeirs* Bones.

Multiplication of *Na-peirs* Bones.

MUltiplication and *Division* are accounted the hardest parts of *Vulgar Arithmetick*; but by the Invention of the Right Honorable *John* Lord *Nepeir* Baron of *Mercifton* in *Scotland*, they are both of them made very easie, and more certain then in the ordinary way, the memory not being charged at all, but as in *Addition* and *Substraction*; what this Artifice is, is shewed by himself in his Treatise of *Rabdologia*, as also in a short Treatise of their Construction and Use by me lately Published. Entituled, *The Art of Numbring by Speaking of Rods*, and therefore it will not be necessary here to make any large discourse of them, it shall be sufficient here, only to give you a description of them, and how to use them.

These Rods or Bones are nothing else, but the ordinary *Multiplication Table*, commonly called *Pythagoras* his Table, cut into pieces, with Diagonal lines between the figures of every product in single *multiplication*; as by the figure of them following it doth appear.

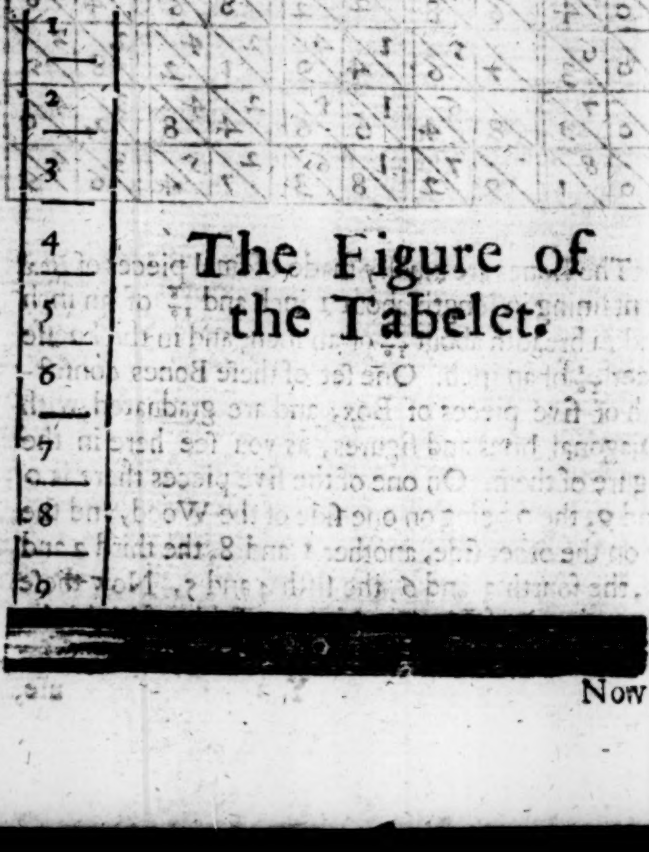
A Figure of the Bones.

0	9	1	8	2	7	3	6	4	5
0	1/8	2	1/6	4	1/4	6	1/2	8	1/0
0	2/7	3	2/4	6	2/1	9	1/8	1/2	1/5
0	3/6	4	3/2	8	2/8	1/2	2/4	1/6	2/0
0	4/5	5	4/0	1/0	3/5	1/5	3/0	2/0	2/5
0	5/4	6	4/8	1/2	4/2	1/8	3/6	2/4	3/0
0	6/3	7	5/6	1/4	4/9	2/1	4/2	2/8	3/5
0	7/2	8	6/4	1/6	5/6	2/4	4/8	3/2	4/0
0	8/1	9	7/2	1/8	6/3	2/7	5/4	3/6	4/5

The Bones are usually made of smal pieces of *Box* containing in length about 1 inch and $\frac{5}{10}$ of an inch and in breadth about $\frac{2}{10}$ of an inch, and in thicknesse neer $\frac{1}{10}$ of an inch. One set of these Bones consisteth of five pieces of Box, and are graduated with Diagonal lines and figures, as you see here in the figure of them. On one of the five pieces there is 0 and 9, the 0 being on one side of the Wood, and the 9 on the other side, another 1 and 8, the third 2 and 7, the fourth 3 and 6, the fifth 4 and 5. Now these five pieces of Box contein one Table of Multiplication cut in pieces, but so few are not sufficient for

use, for you cannot well have lesse then six sets of them, that is 30 pieces, and so many are usually made.

For the placing of these Rods or Bones, when you use them, there belongeth a piece of Wood about four inches in length, and two inches in breadth, having 2 ledges of the same thicknesse with the Rods or Bones, on the thin ledge which is at the end of this board, there is figured the nine Digits, 1, 2, 3, 4, 5, 6, 7, 8, 9. This board is called a Tabelet, because we usally say when the bones are laid thereon for any use the Bones are Tabelated. See the figure thereof.



The Figure of the Tabelet.

Now

Now these *Bones* thus ordered and Tabellated do not only give you the product of every single Digit, multiplied by it self, or any one of the Digits, but the several products of all the 9 Digits together, by every of the 9 Digits, as if it were demanded how much 123456789, will amount unto, being multiplied by 6, in this Table it will be found to be 740740734, for the product of 9 multiplied by 6 is 54, the first figure whereof towards the right hand *viz.* 4 is placed under the Diagonal line, and 5 being in the place of tens is placed above it, again 8 multiplyed by 6 is 48, the first figure whereof towards the right hand, *viz.* 8. is placed under the Diagonal line as in the other product, and these 2 figures 5 in the upper part of the former product, and 8 in the lower part of this must be added together, and being so they make 13, therefore next unto 4 in the first product, I set down 3, and carry 1 to 4 the figure placed above the Diagonal in this second product, which together with the lower figure in the next product, *viz.* do make 7, for 4 and 1 that I carried is 5 and 2 the figure under the Diagonal in the next product are 7, and so the 3 first figures of this product towards the hand are 734, in like manner must the figures remaining between the Diagonals be added together, and their sum if it amount not to ten, must be set down in his proper place, but if they make just 10, or exceed ten, set down the overplus, and carry the ten in mind to the next figures between the Diagonals as in the last product; So likewise in that which follows, 4 and 6 the figures between the Diagonals being 10, I set down a Cypher, and carry one to the next figures, which

which are 3 and 0, now 3 and 1 that I carried is 4, and 4 and 0 is 4 still, and therefore next to the Cypher I set down 4 in the paper; the figures in the next Diagonal are 3 and 4, that is 7; wherefore I set down 7 in the paper; the next are 2 and 8 that is 10 therefore I set down 0 in the paper and carry 1, the next are 1 and 2 which with 1 that I carried make 4, therefore I set down 4 in the paper, the last are 1 and 6 which make 7 and so at last the whole product of 123456789 multiplyed by *6* is 740740734.

Now as the multiplication Table thus ordered doth give the product of all the *9* Digits placed orderly by every of the *9* Digits, so being cut in pieces or slips, and having many of them made in Pastboard of Wood, any other sum may be placed on the Tabellat, and the several products thereof by every of the *9* Digits will be found in the like manner; Thus if it were required to multiply this number 5767 by every of the *9* digits, placing the Rods which have these figures on the top orderly as they are in the following Figure, the Product of 5767 by 3 will be found to be 17301, the product thereof by 8 is 46136, and so for any other.

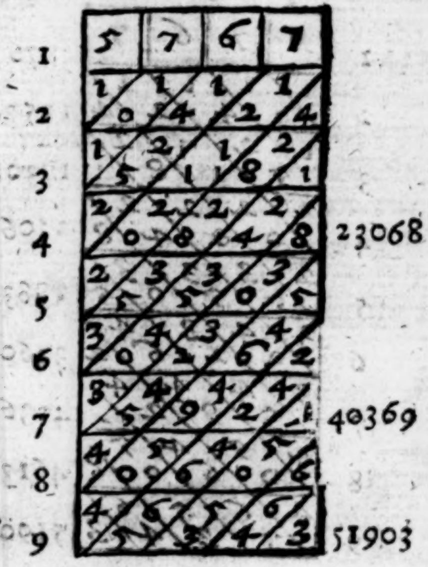
Also

1	5	7	6	7	5767
2	1/0	1/4	1/2	1/4	11534
3	1/5	2/1	1/8	2/1	17301
4	2/0	2/8	2/4	2/8	23068
5	2/5	3/5	3/0	3/5	28835
6	3/0	4/2	3/6	4/2	34602
7	3/5	4/9	4/2	4/[illegible]	40369
8	4/0	5/6	4/0	5/6	46136
9	4/5	6/3	5/4	6/3	51903

Alſo if it were required to multiply 5767 by 749, the product of 5767 by 9 is 51903, and the product thereof by 4 is 23068, and the product by 7 is 40369, and theſe products being ſet down and added together, as in ordinary Multiplication, the product of 5767 multiplied by 749 will be 4319483, as by the operation in the margine it doth appear.

```
     5767
      749
   ------
    51903
   23068
  40369
  -------
  4319483
```

Diviſion



Division by the *Rods*.

LEt it be required to divide 4319483 by 5767 placing this Divisor on the Tabelat, as was directed for *Multiplicati n*, the products of your Divisor by every of the 9 Digits is given you, and may be deducted from the Dividend, as been already shewed, and according to the Rules there delivered, I ask how often is 5767 in 43194, and the answer by my Table is 7 times, and therefore I substract 40369 the product of 5767 by 7 from 43194 and

and there rests 2825, which with the next figure in the Dividend, *viz.* 8, is 28258, in which I find 5767 to be conteined 4 times, and therefore I substract 23068 the product of 5767 by 4 from 28258, and there rests 5190, which with the next figure in the Dividend, *viz.* 3, is 51903, in which I find 6766 to be conteined nine times, and therefore I

	Dividend	*Quotient*
Divisor 5767)	4319483	(749
	40369	*Subst.*
	28258	*Remain*
	23068	*Subst.*
	51903	*Remain*
	51903	*Subst.*
	00000	*Remain.*

substract 51903 the product of 5767 by 9, from 51903 the remaining figures of the Dividend, and there resteth nothing, and therefore the quotient is 749, as by the operation it doth appear.

And thus may you do with any other sum (having a competent number of *Bones* or *Rods* ready for the work) without any charge to the memory, which in other wayes cannot be avoided, and by reason of which many mistakes may happen, which are in this prevented. Also by the *Bones* there is this commodity, that when it is doubtful what figure to set in the Quotient, the Bones will certainly direct you, so that you cannot possibly mistake.

These

These *Rods*, or *Bones* (as they are commonly called) are of excellent use in extracting the *Square* and *Cube Roots*, To perform which by Vulgar Arithmetick, already taught in the first part of this Book, and therefore I shall not insist upon it in this place, but refer the Reader to my forementioned *Treatise*, which shews both the *Fabrick* and *Use* of these *Rods* or *Bones*, not only in *Multiplication* and *Division*, but in the *Extraction* of the *Square* and *Cube Roots*, Which Treatise is entituled, *The Art of Numbering by speaking Rods.*

Having done with the four Species of Arithmetick in this Instrumental way, I should now proced to the *Golden Rule* of *Three*, and consequently to the other Rules of *Arithmetick*, but having in the former parts of this Book performed that already, both in *whole numbers*, *Fractions*, and *Decimals*, I shall only give you an Example in the most necessary Rules, and so conclude this third Part.

But before I cease speaking farther of *Instrumentall* practices, (although I have already taught how to *Extract* the *Square* and *Cube Roots* by *Vulgar Arithmetick* in this *Book*, and by *Napeirs Bones* in the before recited *Treatise*, yet to make this *Instrumentall* part the more compleat, I shall here by a new Artifice shew.

7
7

1. How to find the *Square* or *Cube Root* of any *Number.* And 49

21
21

2. Any *Root* being given, to find the *Square* or the *Cube number* of that Root. 21

42
441
21

441
882

And both these by *inspection* onely, without the help of either *Pen, Compasses,* or any other *Motion.*

9261

For the effecting hereof, there is now inserted, among the forementioned *Decimal Scales* of *Money, Weight, Measure,* &c. namely, between the Scales of *Avoirdupois little weight*, and that of *Dry Measures,* Two other *Scales,* one having written at the beginning thereof the word *Square*, and to the other there is added the word *Cube*, and between them, there is a third line, which hath written upon it, the word *Root*. And by these three *Scales* thus united, the *Square* and *Cube Roots* of any number may be extracted by inspection onely. For

If you find any number whose *Square Root* you require, in the *Line* or *Scale* of *Squares*, right against

against it, in the *Line* of *Roots*, you ſhall have the *Square Root* of that number. Thus,

If the number 64 *were given, and it were required to find the* Square Root *thereof.*

Find the given number *64*, upon the *Line* or *Scale* of *Squares* (which you may do at the letter *a*) and right againſt it, in the *Scale* of *Roots* ſtands the figure 8, which ſhews, that 8 is the *Square Root* of *64*. And in the ſame manner you may find the *Square Root* of any other number.

For againſt	81	in the Line of *Squares*, you ſhall find	9	in the *Scale* of *Roots*, which is the *Square Root* thereof.
	64		8	
	49		7	
	36		6	
	25		5	
	16		4	
	9		3	
	4		2	

In like manner,

If the number 64 *were given, and it were required to find the* Cube Root *thereof.*

Find the given number 64 in the *Scale* of *Cubes*, (which you may do, by counting the ſame number between the ſecond and third figures of 1 upon the Scale at the letter *b*) and right againſt it, in the *Line* or *Scale* of *Roots* ſtands the figure 4, which ſhews, that 4 is the *Cube Root* of 64. And in the ſame manner you may find the *Cube Root* of any other number.

For

For against

729	in the Scale of Cubes, you shall find	9	in the Scale of Roots, which is the Cube Root thereof.
512		8	
343		7	
216		6	
125		5	
64		4	
27		3	
8		2	

And by this *Artifice*, not onely the *Roots* of direct *Square* and *Cube* numbers may be found but in numbers, that be not directly *Square* or *Cube*, the *Fraction* part of the *Root* is nearly discovered also.

I have hitherto given you Examples in such *Square* and *Cube* numbers, as are common and familiar, and that any man may compute almost by memory; but by these the *demonstration* of the Artifice is discovered, the *Lines* of *Squares* and *Cubes* being onely *Square* and *Cube* numbers transferred to Lines. And now let us proceed to greater Numbers. And

1. *For the*

SQUARE ROOT.

In the *Extraction* of a *Square Root*, it is usual to set a *Prick* under the first figure, the third, the fifth the seventh, and so forwards, beginning from the right hand towards the left. And as many pricks as fall under the *Square number* given, of so many fi-

gures will the *Root* of that number consist: So that if the number given be less then 100, the *Root* shall be onely of one figure, if lesse then 10000, it shall be but two figures, if lesse then 1000000, it shall be three figures, and so forward.

Hence it is, That the *Line* or *Scale* of *Squares* is divided first into 100 parts, and if the number given be greater then 100, the first division (which is the place where the first figure of 1 standeth, and which before did signifie *One*,) must now signifie 100, and the whole line shall be 10000; If farther, the number be greater then 10000, you must count or esteem the first figure of 1 to be 10000, and then will the whole line be 1000000 parts; and if this be too little to express the number given, you may proceed farther, and call the first 1. 1000000, and so increase the Line 100 times more; but this is sufficient.

Thus when any Square number is given, if you set it down, and prick it, (or imagine it so to be) If the last prick to the left hand shall fall under the last figure, (which will alwayes be when the figures in the given number be odd) you must find all such numbers upon the line, between the two figures of 1. ——— But if the last prick shall fall under the last figure but one of the given number, (which it will alwayes do, when the figures of the number given are even) then you must find the number given in the line of *Squares*, between the second figure of 1 and 10 at the end of the line.

This being considered, find the number given, whose Square Root is required in its proper place upon the Line of Squares, and against it in the Scale of

of Roots you shall have its Square Root desired. Thus

The *Square Root* of	36	will be found to be	6
	360		19
	3600		60
	36000		189.

And in this manner may the nearest Root of any number not exactly Square be obtained. For

The nearest *Root* of	725	will be found to be about	27
	7250		85
	72500		269
	725000		851

And thus on the contrary, a Number may be *Squared*, as may partly appear by what hath been before delivered, for if you find the *Root* in the *Scale* of Roots, you have its *Square* in the line of *Squares*, and so,

Against	27	in the Scale of *Roots*, you shall find	725	the Square thereof.
	85		7250	
	269		72500	
	851		725000	

Thus much for the Square Root. Now

2. For the
CUBE ROOT.

In the *Extraction* of the *Cube Root*, it is usual to set pricks under the first figure, the fourth, the seuenth, the tenth, and so on, pricking alwayes the third figure from the right hand towards the left. And as many pricks as fall under the *Cubick* number given, so many figures shall be in the *Root*. So that if the number given be less then 1000, the Root shall consist onely of one figure; If less then 1000000, it shall be onely two figures; if it be above 1000000, and less then 1000000000, it will be onely three figures.

Hence it is, That the *Line of Cubes* is divided first into 1000 parts; And if the number given be greater then 1000, the first figures of 1 (which before did signifie onely *One*) must now signifie 1000, and the second figures of 1, must now signify 10000, and the third 1, must signifie 100000, and the whole line must be esteemed to be 1000000. Farther, If the number given be greater then 1000000, the first 1. must signify 1000000, the second 10000000, the third 100000000, and the whole line 1000000000 parts. And if these be yet too little, you may proceed farther, but let these suffice.

Thus when any Cube number is given, if you set it down, and prick it; If the last prick to the left

left hand shall fall under the last figure, the number shall be reckoned between the first and second figures of 1, and the first figure of the Root shall be alwayes either 1 or 2 —— If the last prick fall under the last figure but one, the number given must be reckoned between the second and third figures of 1, and the first figures of the Root shall alwayes be either 2, 3, or 4. —— But if the last prick shall fall under the last figure but two, then the number given must be reckoned between the third figure of 1, and 10 at the end of the Line.

This being considered, find the number given, whose *Cube Root* is desired, in the proper Section upon the *Line* or *Scale* of *Cubes*, and right against it in the *Scale* of *Roots*, you shall have its *Cube Root* desired. Thus

849	The Cube Root of	8490000	will be found to be about	204
849		84900000		439
849		849000000		947

And the like of any other.

On the Contrary, a number may be *Cubed*; for if you find the number in the line of *Roots*, you shall have the *Cube* thereof right against it in the *Scale* of *Cubes*, giving the true denomination to the Cube, according as the part of the Line against which the *Root* standeth doth require.

Thus have you by this Instrumental way of working these things which in the ordinary course are most hard and intricate, rendred very familiar and easie; And although at all times you do not make

uſe of them, yet they are ready helps to confirm you in your working without the tedious way of proving by reverſe working. And here by the way I muſt

Advertiſe.

That the forementioned *Decimal Scales*, and theſe *Lines* of the *Square* and *Cube Roots* are made in Silver, Braſs or Wood, by Mr. *Walter Hayes* at the *Croſſe-Daggers* in *Moor-Fields*, who alſo maketh all ſorts of *Mathematical Inſtruments*.

Example in the Rule of Three Direct.

IF 37 *Ells and a half of linnen Cloth, coſt* 24 l. 7 s. 9 d. *what ſhall* 283 *Ells and an half coſt?*

Firſt, ſet down your 37 Ells, then if you look in your Scale for your half Ell, you ſhall find it to ſtand againſt 500 in the Scale of 1000, which 500 may be called 5 onely, for the two Cyphers may be omitted. Then ſet down your 24 *l.* and for your 7 ſet down 3 decads, which is 6 *s.* and look in your Scale for the Decimal of 1 *s.* 9 *d.* which you ſhall find to be 875; Laſtly, ſet down your 183 Ells, and for your half Ell ſet down 5 as before, ſo will your numbers ſtand thus,

Ells

Ells *li.* *Ells*

If 37.5 cost 24.3875, what 183.5

183.5

1219375

731625

1951000

243875

4475.10625

37.5) 4475.10625 (119.3361

or

375 (119 *li.* 6 *s.* 8 *d.* 2 *q.* $\frac{250}{1000}$

725

375

501

3375

1260

1125

1356

1125

2312

2250

625

375

250

Your numbers thus taken out of your Scale, and placed as here you see, if you multiply the second and third together, you shall find the product of that multiplication to be 4475.10625, which divided by the first number 37.5, giveth in the quotient 119.3361, which is 119 pounds, 3 decads, or 6 shillings, and 361, which reduced by the Scale, giveth 8 pence 2 farthings, and something more.

Example in the Rule of Three Reverse.

IF *when the price of a Quarter of Wheat is* 1 li. 5 s. 6 d. *the peny white loaf shall weigh* 12 *ounces* 16 *peny weight; I demand what the peny white loaf shall weigh, when the price of the Quarter of Wheat is* 7 s. 6 d?

If you place your numbers according to the tenor of the Question, they will stand as followeth.

li. *s.* *d.* *ou.* *pw.* *s.* *d.*
1—5—6—12—16—7—6

But being taken out of the Scales of Money and Troy Weight, they will stand thus,

li.

li.	*ounces*	*s.*
1.275	——12.8——	.375

```
 1.275
 12.8
 -----
 10200      .375) 16.3200(435
 2550             ...          or
1275             ------
-----            1500      ounces pw.
16.3200          1320      43—10 75
                 ----
                 1125
                 1950
                 ----
                 1875
                   75
```

Here if you multiply 1.275 which is the Decimal of 1 *li.* 5 *s.* 6 *d.* by 12.8 which is the Decimal of 12 ounces 16 peny weight, you shall find the product of that multiplication to be 16.3200, which being divided by .375, which is the Decimal of 7 *s.* 6 *d.* the quotient will be 43.5, which is the Decimal of 43 ounces, 10 peny weight 3 grains, and so much ought the peny white loafe to weigh, when the quarter of Wheat is sold for 7 *s.* 6 *d.*

Example

Example in the Double Rule of Three.

IF 24 yards of stuff of three quarters broad, cost 4 l. 14 s. what shall 328 yards of the same stuffe cost being 5 quarters broad.

If you place your numbers according to the directions of this Rule, they will stand thus,

	yards		*quarters*		*li.*	*s.*		*yards*		*q.*
If	24	of	3	cost	4	14,	what shall	328	cost of	5

But if you take your fraction numbers out of their proper Scales, they will stand thus,

yards	*quarters*	*li.*	*yards*	*quarters*
24	3	4.7	328	5

```
24———3———4.7—328——5
 3            4.7
——           ————
72           2296
            1312
           —————
           15416
               5
           —————
         7708.00
```

72) 7708.000 (105055

72 or
508 *li.* *s.* *d.*
107—1—1
504
400
360
400
360
40

First, multiply the two first numbers; as 24 and 3 together, they make 72 for Divisor, then multiply 4.7, which is the Decimal of 4 *l.* 14 *s.* by 328, and the Product is 15416, which again multiplied by 5 the last number giveth 77080, unto this Product, (that there may be a competent number of figures in the quotient,) I add two Cyphers, making it 7708000, which I divide by 72, and the quotient is 107.055, which is 107 *l.* 1 *s.* 1 *d.* and so much is 328 yards of stuffe worth, being 5 quarters broad.

Example

Example in Barter.

T*Wo Merchants having two several Commodities, are willing to barter, or exchange the one with the other. The one hath Indigo, which he will sell at* 4 s. *the pound, for ready money, but in Barter he will have* 4 s. 9 d. *the pound; the other Merchant hath Kersies, which for ready money he will sell for* 3 s. 6 d. *the yard. Now the question is, at what price he must rate his Kersies in Barter, to equalize the* 9 d. *advance upon the pound of Indigo.*

The tenor of the Question is this.

If 4 s. *in Barter require* 9 d. *what shall* 3 s. 6 d. *require?*

Your numbers placed will stand thus,

s.	*d.*	*s. d.*
4 ——	9 ——	3 6

But being taken out of your Scale, they will stand thus,

Decads

Decades	*d.*	*s.*
2	.375	1.75

```
        .375
        1.75
        ----
        1875
       2625
       375
       ----
2) .656|25 (.328
   .....
   ---------
     65
      4
     --
     16
```

Say then by the Rule of Three Direct, If 2 decads or 4 *s.* in Barter require .375, which is the Decimal of 9 *d.* what shall 1.57 require? which is the Decimal of 3 *s.* 6 *d.*

First, multiply .375 by 1.75, the product is .65625, but being it is a fraction, I cut off the two last figures, because we require onely three figures in the quotient, which divided by 2, giveth in the quotient .328, which is the Decimal of 7 *d.* 3 *q.* this 7 *d.* 3 *q.* added to this 3 *s.* 6 *d.* maketh 4 shillings 1 peny 3 farthings, and so much ought he to rate his Kersies at by the yard in Barter to save himself harmless.

Example

Example in Fellowship.

THree *Persons* A,B, *and* C. *bought* 4000 *Sheep*, *which cost* 483 li. 6 s. 8 d. *of which money* A. *paid* 203 li. B. *paid* 165 li. 6 s. 8 d. *and* C. *paid* 114 li. 11 s.

First, say by the Rule of Three Direct.

1 If 483 *li.* 6 *s.* 8 *d.* buy 4000 sheep, how many sheep shall 203 *l.* (which is A. share) buy? *Answer* 1680.

2 Say, if 483 *li.* 6 *s.* 6 *d.* buy 4000 sheep, how many sheep shall 165 *li.* 15 *s.* 8 *d.* (which is B. share) buy? *Answer* 1372.

3 Say again, if 483 *li.* 6 *s.* 8 *d.* buy 4000 sheep, how many sheep shall 114 *li.* 11 *s.* (which is C. share) buy? *Answer* 918.

Your numbers being taken out of your Scale, proceed as followeth.

First for A.

li. *sheep* *li.*
If 483.3333 *buy* 4000, *what* 103
4000
812000

483.3333) 8120000000 (1680

4833333
32866670

28999998
38666720

38666664
00000560

Secondly for B.

li. *sheep* *li.*
483.3333 — 4000 — 165.7833
4000
663133300

483.3333) 6531332000 (1372
4833333
17979990
14499999
34800010
33833331
9666790
9666666
124

Thirdly for C.

li.	sheep	li.
483.3333	4000	114.55

114.55
4000
458200.00

483.3333) 458200000 (948
43499997
23200030
19333332
38666980
38666664
316

The

The manner of Work.

For A. multiply 203 *l.* (which is A. ſhare) by 4000 (which is the number of ſheep bought) and the product is 812000, which number ſhould be divided by 483.3333, but being it is greater then 81200, I therefore add four Cyphers thereto, that I may have four figures in the quotient, and it makes 8120000000, which divided by 483.3333 giveth in the quotient 1680, and ſo many ſheep belong to A.

2 For B, multiply 165.7833 (which is the Decimal of B. ſhare) by 4000, (the number of ſheep bought) and it produceth 663133 2000, which divided by 483.3333, giveth in the quotient 1372, and ſo many ſheep belong to B.

3 For C, multiply 114.55, (which is the Decimal of C. ſhare) by 4000, (the number of ſheep bought) it produceth 4582000, which number ſhould be divided by 483.3333, but being it is not large enough to give figures enough in the quotient, I therefore add two Cyphers, making it 458200000, which divided by 483.3333, giveth in the quotient 948, and ſo many ſheep ought C. to have.

Now for proof, if you add the number of ſheep that A, B. and C. ſhould ſeverally have, you ſhall find them in all to make 400, which demonſtrates the Work to be true.

A	1680
B	1372
C	948
	4008

Examples in Losse and Gain.

IF *one yard of Stuffe cost* 6 s. 8 d. *and I sell the same again for* 8 s. 6 d. *what shall I gain in laying out* 100 li. *upon such a Commodity?*

Take the difference between the price that your Commodity cost, and the price for which you sell it, that is, in this Example, the difference between 6 *s.* 8 *d.* and 8 *s.* 6 *d.* which is 1 *s.* 10 *d*, then say by the Rule of Three Direct.

If 6 *s.* 8 *d.* gain 1 *s.* 10 *d.* what will 100 *li.* gain?

If you place your numbers according to the Rule of Three Direct, as they are here given, they will stand thus,

s. *d.*	*s.* *d.*	*li.*
6 8——	1 10——	100

But being taken out of your Scale and placed, they will stand as followeth.

.3333

s. *s.* *li.*

.3333 ——— .425 ——— 100

100

.3333) 42500 | 00 (127.5

3333

9170

6666

25040

23331

17090

16665

425

Your numbers being placed, multiply .425, which is the Decimal of 1 *s*.10 *d*. by 100 *li*. and the Product is 42500, to which I add two Cyphers (that I may have a competent number of figures in the quotient) and it makes 4250000, which divided by .3333 the decimal of 6 *s*. 8 *d*. giveth in the quotient 127.5, which is 127 *li*. 5 livers or 10 *s*. so there is 17 *li*. 10 *s*. gained in laying out of 100 *li*.

I will here prove this question by the converse.

If by one yard of Stuffe which is sold for 8 s. 6 d. *there was gained* 27 li. 10 s. *in laying out of* 100 li. *I demand what the said stuffe cost a yard at the first hand?*

Add 100 *l.* and 27 *l.* 10 *s.* together, and they make 127 *li.* 10 *s.* Then say by the Rule of Three Direct,

If 127 *li.* 10 *s.* give 100 *l.* what shall 8 *s.* 6 *d.* give?

Take your numbers out of your Scale, and place them as here you see.

li. — *li.* *s.*
If 127.5 gives 100, what .425

127.5) 425000|0 (3333

3825
4250

3825
4250

Here if you multiply 425, which is the Decimal of 8 *s.* 6 *d.* by 100, you shall have 42500, to which if you add a Cypher, you make it 42500.0, this number being divided by 127,5, which is the Decimal of 127 *li.* 10 *s.* giveth in the quotient 3333, and if you had added more Cyphers to the Dividend, you should have had more threes in the quotient, and no other figures, but these four threes are enough, and are a Decimal fraction representing 6 *s.* 8 *d.* and so much did the yard of Stuffe cost at the first hand.

Examples

Examples in Loss & Gain upon Time wrought by the Double Rule of Three.

IF *one Ell of Lockeram cost me* 2 s. 8 d. *ready money, and I sell the same again for* 2 s. 10 d. *the Ell to be paid at the expiration of three moneths; I demand what I shall gain in* 12 *moneths, laying out* 100 li. *upon that Commodity.*

This and such like questions, although they may be wrought by the Rule of Three Direct, at two operations, yet they are best performed by the Double Rule of Three compounded of five numbers, wherefore the question may be thus stated.

If 2 *s.* 8 *d.* in three moneths, gain 2 *d.* what shall 100 *li.* gain in 11 moneths?

If you take your numbers out of your scale, and place them according as was directed in the first part of this Book, you shall find them to stand thus,

s.	*mo.*	*d.*		*li.*		*mon.*
If 1.333	*in* 3 *gain.*	83,	*what shall*	100	*gain in*	12

```
 3          100
 -----      -----
 3.999      8300
              12
           ------
           16600
           8300
           ------
3.999) 99600 (25 fere
          ..
         ------
         7998
        19620
         ------
        19995
          375
```

Your numbers being placed according to the tenor of the question, if you multiply 1.333, which is the Decimal of 2 *s.* 8 *d.* by 3 moneths, the product will be 3.999, which must be your Divisor, then multiply .83, which is the Decimal of 2 *d.* by 100 *li.* and it makes .8300, that again multiplied by 12 moneths, giveth for the product 99600 for your Dividend, wherefore if you divide 99600 by 3999, it will give you in the quotient 25 almost, which is 25 *li.* for the Decimal fraction remaining is so small, that it wanteth not near a farthing of 25 *li.* and therefore we call it 25 *li.* and so

so it is exactly, as you may try, if you reduce all the numbers to their least denominations, and work as is before taught in *Vulgar Arithmetick*.

I will prove this question by the converse.

If one Ell of Lockeram cost me 2 s. 8 d. *ready money, for what price shall I sell the same again to be paid at the end of three moneths. So that I may gain* 25 li. *in* 100 li. *for* 12 *moneths?*

Say by the Rule of Three Direct.

If 100 li. *in* 12 *moneths gain* 25 li. *what shall* 2 s. 8 d. *gain in* 3 *moneths?*

If you take your numbers out of your Scale, and place them according to the Double Rule of Three, they will stand as followeth.

---

li.	*m.*	*li.*	*s.*	*mon.*
100	12	25	1.333	3

```
100——12——25——1.333——3
 12              25
————            ————
 200            6665
100            2566
————            ————
1200            33325
                    3
                ————
          1200) 99975 (.83
                ————
                9600
                 3975
                ————
                 3600
                  375.
```

Your numbers being thus placed, if you multiply 100 *li.* by 12 moneths, you shall find the product to be 1200, which is your Divisor. Then multiply 25 *li.* by 1.333, which is the Decimal of 2 *s.* 8 *d.* and the product thereof will be 33325, which multiply again by 3, and the product will be 99975 for your Dividend, this 99975 divided by 1200, giveth in the quotient .83, which is the Decimal of 2 *d*, which 2 *d.* added to 2 *s.* 8 *d.* the price which the Ell of Lockeram cost, giveth 2 *s.* 10 *d.* and at that price must you sell the same at 3 moneths time, so that you may gain 25 *li.* in the 100 *li.* in 12 moneths.

I

I might further proceed to ſhew you Examples in divers other Rules; As in *Alligation*, *Poſition*, &c. but thoſe Rules being already handled in the Firſt Part of this Book, it will be eaſie to apply them to the ſcales. And (as I intimated at the beginning of this Third Part,) that although I have onely made choice of the eight *Scales* there mentioned and deſcribed, yet it was eaſie to contrive *Scales* for the *Coins*, *Weights* and *Meaſures* of other *Countries*, and not onely ſo, but when the value of the *Money*, *Weight* or *Meaſure* of one *Country*, and the *Money*, *Weight* or *Meaſure* of another Country is known, it is eaſie to contrive two *Scales*, which facing one another, ſhall immediately tell you how many *Pounds* in one place ſhall be equal to ſo many *Crowns*, or other *Coin* in another place; but this I do onely intimate, that ſuch as are deſirous, may fit themſelves with ſcales anſwerable to their moſt neceſſary occaſions. And thus ſhall I conclude this third part, referring ſome things neceſſary to ſuch a Work, to the Appendix following.

The End of the Third Part.

AN ABRIDGEMENT OF THE PRECEPTS OF ALGEBRA.

The Fourth Part,

Written in French.

By

JAMES de BILLY.

And now tranſlated into Engliſh.

With divers Queſtions added, which were not in the Original.

Publiſhed by Will. Leybourn.

LONDON, Printed *Anno,* MDCLXVIII.

ALGEBRA BREVIS.

The Fourth Part.

I Have been alwayes of opinion, that the practice of *Algebra* ſhould not be entangled with great number of precepts : This Science is of it ſelf dark enough, without adding unto it new obſcurity, by the confuſion of many different operations. You have here an Abridgement which hath pleaſed many of good judgements, and I hope, ſuch as will with attention read it, ſhall from thence receive both ſatisfaction and profit.

I ſhall in the firſt place ſet forth a Table of three rancks : In the firſt of which there is a progreſſion natural, of which the terms are diſpoſed in that order, that immediately under them you have the Coſſick Characters, of which they are Exponents, and in the loweſt Cell, a progreſſion Geometrical, which beginning with an Unite, may be doubled, trebled, quadrupled, &c. we have for the greater eaſe only doubled them. Obſerve then that R is a Coſſick Character ſignifying a Root, and that the Exponent

Exponent thereof is marked above in the uppermoſt ranck, and C is the Coſſick Character of a Cube, whoſe Exponent is 3, and ſo of the reſt. I call thoſe terms, which are in the upper Cell, Exponents; becauſe they expound the Coſſick Characters, and the Numbers of the Geometrical Progreſſion which are below. Mark well this manner of ſpeaking, conſider diligently this Table & for the preſent content your ſelf with this.

Exponents.	0	1	2	3	4	5	6	7	8	&c.
Coſſick Characters.	N	R	Q	C	QQ	S	QC	S2	QQQ	&c.
	Numbers abſolute	Root	Square	Cube	Biquadrate	Surſolide	Square Cube	Solide ſecond	Square ſquare ſquare	&c.
Progr. Geometr.	1	2	4	8	16	32	64	128	256	&c.

You may continue this Table, if you pleaſe, infinitely in this manner. Take two numbers, which if you multiply, they will produce ſome Exponent, you ſhall preſently ſee what a character is to be put under that Exponent. For example, If you would know the character of the Exponent 6, take the numbers 2 and 3 (becauſe theſe multiplied together make 6) after

after that add their characters which are Q and C, and you shall have Q C for the character of the Exponent 6. In like manner the character of the Exponent 8 is Q Q Q, because under the exponents 2 and 4, which multiplied together produce 8, are contained the characters Q and Q Q. Also the character of the exponent 12 will be Q Q C, because 12 is a number produced by the multiplication of 2 and 6, or of 3 and 4.

But if the exponent be a first number (that is to say, a number not produced by the multiplication of any two other) mark in what order it is after the exponent 5, and call it sursolide second, third, or fourth, *&c.* according to its ranck. The character of 5 is solid, of 7 solid second, of 11 solid 3; and so consequently under such exponents as are first numbers, under which only are found such solides.

CHAP. I.

The Alegorithm of Cossick Numbers, *simple, compounded, or diminished.*

BY the word *Alegorithm*, I mean all the operations comprehended under these four kinds, *Addition*, *Substraction*, *Multiplication*, and *Division*. By the word *Cossick simple*, I understand such as have not this + (which signifies *plus*, nor this — signifying *minus*, expressed before them. On the other side, by *Numbers composed*, are meant such as have

have the ſign +, and by diminiſhed, ſuch as have the ſign —. Note ſuch numbers as have no ſign expreſſed, are ſuppoſed to have this of +.

Sect. 1. Addition *of ſimple Coſſick Numbers.*

All ſimple coſſick numbers are of the ſame denomination (that is to ſay, have the ſame character) or of different: If they be of the ſame denomination, the Addition is as in common Arithmetick *Example*, 5 Q added with 3 Q, makes 8 Q.

If they be of different denominations, they muſt be added by the interpoſition of the figure +, as *6* R added to 4 Q makes *6* R + 4 Q in like manner 3 added to 4 R makes 3 + 4 R.

Sect. 2. Subſtraction *of Coſſick numbers.*

Either they are of the ſame, or different denominations: if of the ſame, you muſt ſubſtract as in ordinary Arithmetick; for example, 3 Q ſubſtracted from 8 Q, there reſts 5 Q.

If of different, you muſt ſubſtract by the interpoſition of the ſign — as *6* R ſubſtracted from 4 Q, there reſts 4 Q — *6* R, ſo 3 C ſubſtracted from *66*, there remains *66* — 3 C.

Sect. 3. Multiplication *of ſimple Coſſick numbers.*

You muſt here have regard both to the abſolute number, and to the Coſſick characters: If therefore a Coſſick number be to be multiplied by an abſolute, you muſt multiply the abſolute numbers, and unto

unto the product give the ſame character, as 5 R. multiplied by 12 produce 60 R.

But if you multiply Coſſick numbers by Coſſick, you muſt multiply the abſolute numbers together, and to the product give the Character of that exponent, which is made by the addition of the exponents belonging unto the foreſaid Coſſick characters. For example, 2 R multiplied by 3 Q make ſix C, becauſe the exponent of Q is 2 added to one the exponent of R, make 3 the exponent of C, which in this reſpect ought to be given to the product. In like manner 5 R multiplied 4 C it makes 20 QQ for the reaſon above given.

Sect. 4. Diviſion *of ſimple Coſſick numbers.*

The ſpeculation of this is marvellous. But the practice of it is by putting the Diviſor under the number to be divided, drawing between them a little ſmall line in manner of common Fractions. For example, 13 Q divided by 7 R, the quotient is $\frac{13q}{7R}$ and 6 QQ divided by 5 C the quotient is $\frac{6qq}{5c}$.

Sect. 5. Addition *of numbers compoſed and diminiſhed.*

Some order is to be obſerved in this, in which the numbers are to be diſpoſed in ſuch manner, that thoſe that are of the ſame denomination muſt be put right under one another. After you have done this if they have the ſame ſign, they are added as in common Arithmetick, and to the product give the ſame ſign. As for example, 7 Q — 4 C added to 3 Q — 2 C, the Sum is 10 Q — 6 C.

But if the numbers be of different ſigns, the leſſer muſt be ſubſtracted from the greater, and to the reſidue you muſt give the ſign of the greater number as 6 Q + 7 R added with 7 Q — 12 R, give for the Sum total 13 Q — 5 R.

6 Q + 07 R	6 Q — 07 R
7 Q — 12 R	7 Q + 12 R
13 Q — 05 R	13 Q + 05 R

Sect. 6. SUBSTRACTION *of numbers compoſed and diminiſhed.*

There is nothing more intricate to beginners than the precepts commonly given for ſubſtraction. You have here an order plain, ſure, and very eaſie to practice.

Change the ſign of the particulars of that number you deſire to ſubſtract, and after this change add them with the number from which the ſubſtraction is to be made, and you ſhall have the reſidue: As if from 6 Q — 10 R you would ſubſtract 18 Q — 15 R by the 5 Sect. the reſidue will be 5 R — 12 Q. So likewiſe if you would ſubſtract — 8 R — 9 Q from 16 R + 6 Q the reſidue will be 24 R + 15 Q.

6 Q — 10 R	Add	16 R + 6 Q	Add
— 18 Q + 15 R		8 R + 9 Q	
— 12 Q + 5 R	Reſid.	24 R + 15 Q	Reſidue

Sect. 7.

Sect. 7. Multiplication *of numbers composed and diminished.*

Mark what I said of Cossick simple in the 3 Sect. and remember that the same signs have alwayes the sign ✠ in the product & different — & there is no difficulty in multiplication, so as you multiply every particular of the multiplicand by every particular of the multiplicator, as in common Arithmetick, as if you multiply 3 Q — 2 R by 8 R, 2 R by 8 R, it makes 16 Q and because the multiplicator & multiplicand have different signs, the product must have the sign of — ; and therefore that shall be 16 Q, farther 3 Q by 8 R make 24 C, to which you must give the sign ✠, because the multiplicator and the multiplicand have the same sign, so that the product of this multiplication will be 24 C — 16 Q. So 2 R ✠ 4 Q multiplied by 3 Q — 5 the product is 6 C ✠ 12 QQ — 10 R — 20 Q

3 Q — 2 R multiplicand	2 R ✠ 4 Q multipl.
8 R multiplicator	3 Q — 5 multipl.
24 C — 16 Q Product.	— 10 R — 20 Q
	6 C ✠ 12 QQ
	6 C ✠ 12 QQ — 10 R — 20 Q

Sect. 8. Division *of numbers composed and diminished.*

There is no great difficultie in this, only put a

line between the Dividend and the Divisor, and you have the quotient, as $4\,C - 3\,Q + 2\,R$ divided by $5\,R - 4\,C$ make for this quotient

$$\frac{4\,C - 3\,Q + 2\,R}{5\,R - 4\,C}$$

Sect. 9. Algorithm *of Fractions.*

I shall not give any particular precepts, because if a man understand the Fractions of common Arithmetick, and practice according to what is before said there will be no need of them.

CHAP. II.

The Rule of Algebra, *with the explication thereof.*

IT was meerly necessary by the precedent rules, to trace out insensibly the way to *Algebra*, which cannot be practiced without *Addition*, *Substraction*, *Multiplication* and *Division*. Having therefore made plain these difficulties, we will proceed in the *proposition* of the Rule of *Algebra*, and the explication of every part of it briefly and plainly.

Sect. 1.

Sect. 1. *The Rule of* Algebra.

You must first for the Number unknown put 1 R, and after examine this root according to the tenour of the question, untill you come to an Equation. Secondly, this Equation is to be reduced, if need require. Thirdly, you must divide every part of the Equation by the number of the greatest cossick character. After which, either the quotient, or some root of the quotient, will give the root unknown. This is the Rule of *Algebra*, let us now explain it.

Sect. 2. *How the* Equation *must be found.*

The rule saith, this is done by examining the question propounded, according to the tenor of the same. That is to say, you must well observe all the conditions of the question propounded, to the end you may fully accomplish it. For after you have gone thorow it, you shall find an equation between two numbers. As if I search a number, which added with its square shall make 20, I suppose this number unknown to be 1 R, the square thereof is 1 Q, (because every number multiplied by it self, makes its square) then 1 Q + 1 R is equal to 20. See thus an equation found between 1 Q + 1 R and 20.

Sect. 3. *How your* Equation *must be* reduced.

Your Equation being found, it is reduced by adding the same number to both the terms of the e-

quation,or by ſubſtracting from them the ſame number. So is it performed by multiplying, or dividing both the terms by the ſame number. For by this means your equation ſhall remain the ſame after theſe things done. As for example, $1\,R + 1\,Q = 20$, adding throughout $2\,C$, you ſhall have alſo an equation between $1\,R + 1\,Q + 2\,C$ and $20 + 2\,C$. So ſubſtracting $1\,R$ from the terms, you have $1\,Q = 20 - 1\,R$. Likewiſe multiplying, or dividing both your terms by 3, you ſhall have by the multiplication $3\,R + 3\,Q$ equal to 60, and by the diviſion the equation will be between on $\frac{1}{3}\,R + \frac{1}{3}\,Q$ and $\frac{20}{3}$.

Now to make your reduction judiciouſly and profitably, you muſt take care alwayes that your greateſt character remain alone on one ſide of your equation. As of all the reductions before made, there is none uſefull but the ſecond, becauſe in that only you find on the one ſide alone $1\,Q = 20 - 1\,R$ which is the only end of your reduction.

I ſaid in the rule of *Algebra*, that your equation muſt be reduced, if it be neceſſary; becauſe it ſometimes happens, that there is no need of it: as when your equation fals out between two ſimple collateral numbers, I call thoſe numbers collateral, whoſe exponents do not ſurpaſſe one another by more than an unite.

Sect. 4. *When you muſt* extract *the* root.

When your coſſick numbers are ſimple and collateral, you muſt not extract any root; but if you divide by the number of the greateſt coſſick character, the

quotient ſhall ſhew you the value of the root, which is all you ſeek for in *Algebra*. For example, If you find an equation between 2 R and 28, dividing ſimply 28 by 2 the quotient, ſhall be the value of 1 R.

But when the terms of your equation are not collateral, you muſt extract ſome root; either ſquare cube, ſquared ſquare, &c. according to the coſſick character which remains after your Hypobibaſm.

Now Hypobibaſm is nothing elſe but an abatement, or depreſſion of the character, and is done by ſubſtraction of the letter exponent from the greater. As if you find an equation between 10 QC and 90 QQ, take notice of the exponent of QC in the Table inſerted at the beginning, the which exponent is 6; afterwards look to the exponent of QQ which is 4, ſubſtract 4 out of 6 there reſts 2, of which the coſſick character is Q. From hence I conclude, that 10 Q are equal to 90, after dividing 90 by 10, and finding 9 in the quotient, I conclude that the ſquare root of 9 muſt be extracted by reaſon of the character Q.

Sect. 5. *How to* extract *the* ſquare root *of numbers, compound and diminiſhed.*

No man hath yet perfectly found out the way to extract the root of numbers compound and diminiſhed, unleſſe the exponents of three terms of the equation keep between them in ſome ſituation or other, an Arithmetical proportion, that is to ſay, the ſame diſtance. As if the equation be between 1 Q & 20 — 1 R, you may now extract the root of 20 —

 1 R,

1 R, because the exponents of the three numbers which make the equation, are 2, 0, 1, which thus place, 0, 1, 2, keep the same distance.

The greatest cossick character left after the Hypobibasm, shews the root to be extracted; as in this example, before the square root is to be extracted, because the greatest character is Q. The method to be followed in the extraction, see here propounded in general terms.

Take first the half of the number of roots. Secondly, to the square of this half, add or from it subduct your absolute number according to the sign of ✠ or —. Thirdly, extract the square root of this sum, or of the residue. Fourthly, to this Root add or from it substract half the number of roots, and this last sum or residue shews you the value of the root unknown. For example, I would find a number, the double whereof added to its square, should be equal to 24, I shall find an equation between 2 ℞ ✠ 1 Q and 24 by the second Section. Moreover, I shall reduce this equation after this manner 1 Q equal to 24 — 2 R by the third Section. Then if I divide 24 — 2 R by 1, the number of the greatest cossick character there still remains 24 — 2 R, because an unite doth neither multiply nor divide. Then in as much as the three terms of the equation do keep an Arithmetical proportion, I extract the square root of 24 — 2 R in this manner. I take first half the number of roots, which is 1. Secondly, the square of 1 is 1, to which I add the absolute number, which is 24, because of the sign ✠ before it, that makes 25. Thirdly, I extract the square root of 25, which is 5. Fourthly, from this root

root I extract the moiety of the number of roots which is 1, because of the sign — the residue will be 4; whence I conclude that to be the value of one root, and that the number sought is 4, whose double is 8, added to the square (of 4,) 16, makes 24.

Here note that numbers diminished, where the absolute number hath the sign —, have two roots. The greater is extracted, as before we have taught; the lesser is found out by substracting the square root of the residue from the half sum of the roots. As if I seek a number whose octuple diminished by 12, shall be equal to its square, you will find an equation between 1 Q and 8 R — 12. The greatest root is 6, the lesser is 2, here both the roots answer the question. But this happens not often.

But if you be to extract the biquadrate root: First, extract the square root, as is taught, and again extract the square root of this, and this shall be your biquadrate root. As if the equation be between 1 QQ and 2 Q + 8, you may find the square root to be 4 by the method taught, first, taking half the number of the squares, &c. and afterward you must extract the square root of 4, which is 2, this shall be the value of the root. In like manner, if your equation be between 1 QC and 2 C + 48, first, I extract the square root of 2 C + 28, which is 8, of which extract the cube root 2, because the root to be extracted is the square cube, as the character Q C, which is one of the terms of the equation, denoteth

Sect. 6. *How to know if the* question *be impossible, vain, or ill propounded.*

You may know the question to be impossible, if you

you come to an equation impossible : As if following the conditions of the Probleme, you meet with an equation between 6 R and 24 R, or between 3 Q ✢ 5 and 4 ✢ 2 Q.

Secondly, the question is vain, when the equation is between two equal numbers of the same denomination, as between 6 Q and 6 Q.

Thirdly, the question is ill propounded, when without any difficulty many numbers will answer the probleme propounded.

CHAP. III.

Algorithm of second Roots with their use.

ALgebraists sometimes use more than one root to find out divers numbers propounded, and then to the end they may proceed with lesse confusion they usually help themselves with second roots which they expresse thus, 1 A, 1 B, &c.

Sect. 1. Addition *of second roots.*

If your second roots be of the same denomination add the numbers, and to the sum give the same denomination, as 5 A added to 4 A make 9 A, if they be of different denomination, add them with the sign of ✢ as 5 A added to 6 B make 5 A ✢ 6 B.

Sect.

Sect. 2. Substraction *of second Roots.*

If they be of the same denomination, substract one number from the other, and to the residue give the same denomination, as 5 A taken from 9 A, there remains 4 A, if different, they are substracted with the sign — as 6 B taken from 8 A, there rests 8 A — 6 B.

Sect. 3. Multiplication *of second Roots.*

If they be of the same denomination, do as you do with the first roots, as 4 A multiplied by 7 A make 28 A Q, if of different, both denominations are reteined in the product, as 3 R multiplied by 5 A make 15 R A.

Sect. 4. Division *of second Roots.*

Division is only performed by the interposition of a little line, as is before taught, notwithstanding it is to be observed, that if for example 3 A R be to be divided by the Divisor 1 R the quotient shall be 3 A, because in such case there needs nothing else but to take from the Dividend the character of the Divisor.

Sect. 5. *The extraction and use of second Roots.*

After you have found and reduced your equation, according to the manner of working in second roots, you must extract the root after the manner taught in the

the precedent Chapter: As if 1 A Q be equal to 25, I say that 5 is the value of the second root, & if 1 A Q be equal to 4 A ✠ 12, you must take the moiety of the number of roots, &c. As is said in the 5 Sect. of the precedent Chapter, and you shall find *6* to be the value of 1 A.

Now since the end of the second roots is to be reduced to first, you must not forget after you have found the value to begin again your work, and to put in first roots that which you have found to be the value of the second, as I shall shew you in some examples in a Chapter following.

CHAP. IV.

The Algorithm *and extraction of the Roots of surd and irrational numbers.*

SUrd Roots are those that have a radical sign before them, and which in propriety of speech ought to be called absolute numbers, notwithstanding they cannot be expressed by any common number, neither whole, nor broken, we will hereafter expresse the radical sign by this character ℞.

There are many sorts of surd roots, some are simple, as ℞ Q 5, that is to say, the root square of 5, others are compound, as ℞ Q 5 ✠ ℞ C *6*, that is to say, the root square of 5, *plus* the root cube of *6*,

some

ſome are univerſal, whoſe radical character, extends to all the particulars following it, and for that end are encloſed in a Parentheſis in this manner ℞ Q (14 ✠ ℞ Q 4) the root univerſal of 14 joyned with the root ſquare of 4, all which number is 4 for 14 ✠ the root ſquare of 4 which is 2, maketh 16 whoſe root is 4.

Sect. 1. Reduction *of ſurd roots ſimple to the ſame denomination.*

Firſt, you muſt put the radical ſigns under the numbers to which they belong. Secondly, you muſt multiply the numbers by the ſigns acroſſe, for to get new ones. Thirdly, you muſt add the ſigns together which is done by multiplying their exponents, and give the character of the product common to the two new products, as if you would reduce to the ſame denomination ℞ Q 5 and ℞ C 4, you muſt firſt place them as followeth.

125 16

5 4

℞ Q ℞ C

2 —— 3

6

Secondly, you muſt multiply the numbers 4 and 5 by their ſigns acroſſe, that is to ſay, you muſt take the ſquare of 4, and the cube of 5, which are 16, and

and 125. Thirdly, the exponents of the ſigns ℞ Q and ℞ C, which are 2 and 3, ought to be multiplied together, the product is 6. I look then in the Table what coſſick character is under the exponent 6, and finding QC, I take that for my common denominator, and in ſtead of my two firſt ſurd roots, which were of different denomination, that is to ſay ℞ Q 5, and ℞ C 4. I have two new ones of the ſame denomination, that is to ſay, ℞ QC of 125, and ℞ QC of 16.

Sect. 2. Multiplication *and* Diviſion *of ſurd ſimple Roots.*

If the Roots be of the ſame denomination you muſt only multiply and divide the numbers by themſelves, and to the product and quotient give the ſame radical ſign as ℞ Q 7 by ℞ Q 2 give for the product ℞ Q 14. In like manner ℞ Q 36 divided by ℞ Q 12, gives for the quotient ℞ Q 3.

But if the Roots be of different denomination, you muſt reduce them to the ſame denomination by the precedent Paragraph, and after multiply and divide as ſhall be ſhewed. for example, ℞ Q 3 multiplied by 2, the product is ℞ Q 12, and ℞ Q 12 divided by 2, the quotient is ℞ Q 3.

Sect. 3. *How to know whether two* ſurd roots *be commenſurable or not.*

You muſt divide the greateſt root by the leſſer, if the quotient be rational, the two roots are commenſurable: if otherwiſe, they are not. As becauſe ℞

℞ Q 24 divided by ℞ Q 6, gives for the quotient ℞ Q 4, which is 2 a rational number, I conclude these two roots ℞ Q 24 and ℞ Q 6 to be commensurable. In like manner, since root square 24 divided by ℞ Q 8, the quotient will be 3, a surd number and irrational, you may conclude those two roots, ℞ Q 24, and ℞ Q 8, to be incommensurable.

Sect. 4. Addition *of simple irrational roots.*

If the roots be incommensurable, you must add them only by the sign + as ℞ Q 24 added unto ℞ Q 8 makes ℞ Q 24 + ℞ Q 8. But if they be commensurable, you must add a unite to their quotient rational, and you shall have a sum, which being multiplied by the lesser of the two roots to be added will give a product which shall be the sum sought. As ℞ Q 24 added with ℞ Q 6 makes ℞ Q 54, because ℞ Q 24 divided by ℞ Q 6, gives 2 for the quotient rational, to which I add a unite, and it is 3 by which (alwayes reducing them to the same denomination) I multiply ℞ Q 6, which is the lesser of my two roots, and I find for my summe ℞ Q 54.

Sect. 5. Substraction *of simple irrational roots.*

If they be incommensurable, you must substract them by prefixing the sign — as ℞ Q 8 substracted out of ℞ Q 24, the residue shall be ℞ Q 24 — ℞ Q 8.

But if they be commensurable, you must take away

way a unite out of the quotient rational, and you ſhall have the reſidue, the which being multiplied by the leſſer of the roots given, ſhall give a product which ſhall be the reſidue ſought, as if I be to ſubſtract ℞ Q 6 from ℞ Q 24, dividing the greater by the leſſer, the quotient rational is 2, from which if you take 1, there remains 1, by which (reducing them firſt to one denomination) if you multiply the leſſer root, that is to ſay, ℞ Q 6, the reſidue will be ℞ Q 6.

Sect. 6. Addition *and* Subſtraction *of ſurd numbers, composed and diminiſhed.*

I have here no new precepts, onely advertiſe you, that you may remember what I have ſaid before of Coſſick numbers, touching the ſignes of ✠ and — in the fifth and ſixth Paragraph of the firſt Chapter, and what is delivered in the fourth and fifth of this Chapter, touching the Addition and Subduction of ſimple ſurd numbers, and theſe will be no difficulty, as if you be to add 5 ✠ ℞ q. 24 with 3 ✠ ℞ q. 6, you will find 8 ✠ ℞ q. 54. In like manner, if you ſubduct 3 — ℞ q. 6, from 5 ✠ ℞ q. 24, there reſts ℞ q. 54 — 42.

Sect. 7. Multiplication *of numbers ſurd, compoſed and diminiſhed.*

This Multiplication hath no great difficulty, nor needs new precepts. Remember onely that the ſame ſignes have in the Product ✠ and different — with this, that your Multiplication is not good, if

if the particulars to be multiplied be not first reduced to the same denomination. For example, 5 + ℞ q. 24 by 3 — ℞ q. 6, 'tis to be done after this manner: + ℞ q. 24 by — ℞ q. 6, maketh — ℞ q. 144, or — 12 after + 5 by — ℞ q. 6, maketh — ℞ q. 150. Further, + ℞ q. 24 by 3 is + ℞ q. 216. Lastly, + 5 by 43 is + 15, then the whole Product will be 15 + ℞ q. 216 — ℞ q. 150 — ℞ q. 144, or 3 + ℞ q. 316 — ℞ q. 150 because ℞ q. 144 is a rational number, to wit 12, which being subducted out of 15, because of the signe — leaveth 3.

Example:

5 + ℞ q. 24
3 — ℞ q. 06

— ℞ q. 150 — ℞ q. 144
15 + ℞ q. 216.

15 + ℞ q. 216 — ℞ 150 — ℞ q. 144
3 + ℞ q. 216 — ℞ q. 150

25 9
6 5 24 3
X X
℞ q. 0 ℞ q. 0
℞ q. 150 ℞ q. 216

Sect. 8. *Division of surd numbers, compounded or diminished.*

If the Divisor be simple, the division is made by the interposition of a line between the Divisor and the compound number to be divided, as if ℞ q. 2 + ℞ q. 5 be divided by 8, the quotient will be $\frac{\text{℞ q. } 2 + \text{℞ q. } 5,}{8}$ and so of others.

But because it sometimes may fall out (though very seldom) if the Divisor also will be a Binome, or compound number, that is to say, a surd number compounded of two particulars with the signe +, or a Trinome that is compounded of three particulars, &c. See here the manner to divide in such a case.

If the Divisor be a Binome, you must multiply by his Apotome, as well the number to be divided, as the Divisor (and if the Divisor be an Apotome, you must divide by the Binome, as well the Dividend, as the Divisor) by means of this Multiplication you shall have a new Dividend, and a new Divisor. Now this new Divisor will be alwayes rational, and therefore needs only to be set under the Dividend with a line between. As for example: ℞ q. 6 — 2 by ℞ q. 5 + ℞ q. 3, I take the Apotome of my Divisor (that is) ℞ q. 5 — ℞ q. 3. by which I multiply both my Dividend and my Divisor, by one of the multiplications is produced ℞ q. 30 — ℞ q. 20 — ℞ q. 18 + ℞ q. 12, for my new Dividend, and by the other is produced 2 for my

my new Divisor. So that the quotient of my Division will be ℞ q. 30 — ℞ q. 20 — ℞ q. 18 ✠ ℞ q. 12,

4

because 2 must be first squared, and then 4 put underneath the Dividend.

If your divisor be a Trinome, you must observe the same method, multiplying the dividend and the Divisor by the Apotome of the Divisor, that is to say, by the same Divisor onely, changing the signe of the last particular. After this is done, you shall have a new Dividend and a new Divisor, which shall be a Binome. Then you must again seek a new Dividend and Divisor, which now will be simple and rational.

Last of all, if you will not take this pains, the Division is good, if under the Dividend you subscribe the Divisor with a line between.

Sect. 9. Multiplication *of roots universal.*

You must reduce the root to be multiplied, and the Multiplicator to their squares or cubes, according to the radical signe prefixed, and afterward perform your multiplication, as is taught in Sect. 7. of this Chapter. Afterward you must affix the signe radical, and inclose all in a Parenthesis.

This is better understood by an example, as if you multiply ℞ q. (7 ℞ q. 3) by 2, the squares of the one and other numbers are 7 ✠ ℞ q. 3 and 4, then the first being multiplied by the last, maketh 28 ✠ ℞ q. 48, and therefore if you close this number within a Parenthesis, and put before it the

ſame radical ſigne, the product will be ℞ q. (28 + ℞ q. 48.)

In like manner, if you would multiply this number $\frac{7}{2}$ — $\frac{1}{2}$ R + R q. ($\frac{49}{4}$ — $\frac{3}{4}$ q. — $\frac{7}{2}$ R) by it ſelf, to get the ſquare of it, you muſt call to mind the fourth propoſition of the ſecond Book of *Euclide*, which ſheweth that a line being divided into two parts, the ſquare to the whole is equal to the ſquare of the parts, and to double their Rect-angles, you muſt therefore conceive this number, as divided into two parts, of which the firſt is $\frac{1}{2}$ — $\frac{1}{2}$ R, and the laſt ℞ q. ($\frac{29}{4}$ — $\frac{3}{4}$ q. — $\frac{7}{2}$ R) take then the ſquares of the parts, which are $\frac{49}{4}$ + $\frac{3}{4}$ q. — $\frac{7}{2}$ R, the double of the Rectangle of the parts is ℞ q. ($\frac{9604}{16}$ + $\frac{220}{16}$ q. — $\frac{2744}{8}$ R + 7 C — $\frac{12}{16}$ q. q.) then the ſquare of the number propoſed is the ſumme of theſe three numbers, that is to ſay, $\frac{49}{2}$ — $\frac{1}{2}$ q. — 7 R + ℞ q. ($\frac{9604}{16}$ + $\frac{220}{16}$ q. — $\frac{2744}{8}$ R + 7 C $\frac{11}{16}$ q. q.) The ſquare of the Apotome is the ſame number, putting onely the ſigne — before the univerſal root, and the ſum of the two ſquares is 49 — 1 q. — 14 R.

Sect. 10. Diviſion *of Roots univerſal.*

You muſt reduce the roots to be divided, and the Diviſor to their Squares, Cubes, &c. And after divide them as is taught in the 8 Sect. and when this is done, encloſe all in a Parentheſis, with the ſame radical ſigne which was before. As if you divide ℞ q. (13 + ℞ q. 17) by the root ſquare of 5, their ſquares are 13 + ℞ q. 17 and 5, then the firſt being divided by the laſt, the pro-

duct

duct will be $2\frac{3}{5}$ ✠ ℞ q. $\frac{17}{25}$, then the quotient of the division proposed will be ℞ q. ($2\frac{3}{5}$ ✠ ℞ q. $\frac{17}{25}$.)

Sect. 11. Addition *and* Subduction *of roots universal,*

Many trouble themselves to give precepts intricate enough, the short and most certain is to add them with the signe ✠, and subduct them with the signe —. As for example, ℞ q. (3 ✠ ℞ q. 2) added with ℞ q. (℞ q. 5 ✠ 6) will be ℞ q. (3 ✠ ℞ q. 2) ✠ ℞ q. (℞ q. 5 ✠ 6) and the same first root subducted from the last, the residue is ℞ q. (Æ q. 5 ✠ 6) — ℞. q. (3 ✠ ℞. q. 2.)

Sect. 12. Extraction *of the roots of* Binomes *and* Apotomes.

1 Take the difference of the squares of the one and the other part of the Binome. 2 Add and subduct the square root of this difference from the greatest part of the Binome. 3 Conjoyn the square root of the moiety of the summe, with the square root of the moiety of the residue, by the signe +, if it be a Binome, and by the signe —, if it be an Apotome; and thus the extraction is finished. As if you would extract the square root of this Binome $\frac{3}{2}$ ✠ ℞ q. $\frac{5}{4}$, you shall first take the square of the first part, which is $\frac{9}{4}$, and the square of the second which is $\frac{5}{4}$, the difference of these two is $\frac{4}{4}$, that is to say, 1. Secondly, you must

extract the ſquare root of this difference, which is 1, you ſhall adde and take it away from the firſt part of the Binome, by the addition you ſhall have $\frac{5}{2}$ for your ſum, and by ſubduction $\frac{1}{2}$ for your reſidue. Thirdly, joyn the root ſquare of the ſum, with the root ſquare of the reſidue by the ſign ✠ and you ſhall have ℞ q. $\frac{5}{2}$ ✠ ℞ q. $\frac{1}{2}$ for the ſquare root of your Binome propounded, and conſequently ℞ q. $\frac{5}{2}$ — ℞ q. $\frac{1}{2}$ ſhall be the root ſquare of the Apotome $\frac{3}{2}$ — ℞ q. $\frac{5}{4}$.

CHAP. V.

The Uſe of Algebra.

'TIs much to have taken the pains to learn all that we have hitherto ſhewen or taught: but I dare boldly ſay, that thoſe that ſhall reſt here, do as yet know nothing to the purpoſe, although they may know all the precepts; it behoveth us then to make a ſtep further, to apply and bring thoſe precepts into uſe and exerciſe. 'Tis that which I deſire to demonſtrate in this Chapter, by ſome *Queſtions*, the ſolutions of which will give great light to the attaining of perfection in this Art. Wherefore I intreat thee (Reader) not to omit this Chapter, in which I pretend to yield thee ſome pleaſure and delight, as alſo an illuſtration of what hath been before treated of.

Sect.

Sect. 1. *Questions resolved by one simple* Equation.

Question I.

ALexander *one day told* Ephestion, *That he was elder than him by two years; thereupon* Clitus *tells them, That he was as old as both of them, (their ages added together) and four years over and above. The Philosopher* Callisthenes *being present at this discourse (saith he) I well remember that my father, who was 96 years old, had the age of you three. It is demanded here, how old* Alexander *was when he held this discourse, as also how old* Clitus *and* Ephestion *were.*

I put for the age of *Ephestion* 1 R of years, whence it follows, that *Alexander* had 1 R + 2; therefore *Clitus* had 2 R + 6, and those three together, according to the condition of the question, ought to be equal to 96: therefore there is an equality between 4 R + 8 (which is the sum of the three ages) and 96, take away 8 from both parts of the equation, so there will remain on one side 4 R equal to 88, which divide by the number of the greatest Cossick Character, that is to say by 4, the quotient gives 22 for the value of one root, which was supposed for the age of *Ephestion.* Therefore *Ephestion* was at that time aged 22 years, *Alexander* 24, and *Clitus* 50, which altogether make 96 years.

Question II.

A Hare is 100 *Geometrical paces distant from a Dog that swiftly pursues her, and the Dog runneth two times and an half faster than the Hare: It is demanded how many Geometrical paces the Hare will have run when the Dog overtaketh her.*

I put for these Geometrical paces 1 R; therefore the Dog which runs 100 paces more than the Hare, will have run 100 + 1 R: and for that the Dog runs twice and an half swifter than the Hare, I take two numbers in like proportion to one another, that is to say 5 and 2, and conclude that there is the same proportion between 100 + 1 R to 1 R, as between 5 and 2; therefore the product of the first number 100 + 1 R multiplyed by the last number 2 (which is 200 + 2 R) is equal to the product of the two means 1 R and 5; (which will be 5 R) therefore if you take from both parts 2 R, there will remain 200 by 3, which is the number of the greatest Character, and find in the quotient *66* $\frac{2}{3}$ which will be the value of the root. I say therefore that the Hare will have run *66* Geometrical paces, and $\frac{2}{3}$ when the Dog shall have overtaken her, and the Dog will have run 1*66* $\frac{2}{3}$ paces, which make twice and an half more than *66* $\frac{2}{3}$.

Question

Question III.

The Architect Vitruvius *in his Ninth Book Chap. 3. tells us, That* Archimedes *found the quantity of silver that a Goldsmith had mixed in a golden Crown which he had made for the King* Hiero, *(who was obliged by Vow to present it to the gods) weighing* 100 *pounds. It is demanded by what means* Archimedes *could arrive to the knowledge of that secret.*

The common Opinion is, that he took two masses, the one of gold, the other of silver, which weighed as much as the Crown; afterward he filled a vessel up to the brim with water, which vessel was placed in some great Bason, that the water that should be forced out of the first vessel, might be preserved and not lost. Thirdly, he gently put in the two masses and the Crown, one after the other, into the prepared vessel, taking exact notice of the quantity of water that issued out of the vessel at each time, and concluding from thence, that the Goldsmith had mingled 6 pounds and $\frac{2}{3}$ parts of silver. We will suppose then, that the masse of gold weighing 100 pounds, did cast out of the vessel 60 pounds of water, and that the masse of silver also weighing 100 pounds, cast forth of the vessel 90 pounds of water, and that the Crown cast forth 65 pounds. I put afterward for the silver mixed in the Crown 1 R, and constitute twice the Rule of three after this manner.

If 100 pounds of gold give me 60 pounds of water,

water, how much will 100 — 1 R? and I finde $\frac{6000-60R}{100}$ for my fourth number.

Secondly, if 100 pounds of silver give me 90 pounds of water, how much shall 1 R? and I find $\frac{90R}{100}$. Now these pounds of water $\frac{6000-60R}{100}$ and $\frac{90R}{100}$ added together do make $\frac{6000+30R}{100}$ pounds of water cast out, which ought to be equal to 65 pounds of water, cast forth by the Crown, and therefore if we reduce them, we shall finde 6000 + 30 R equal to 6500 (this reduction is made by multiplying the denominator 100 by 65, for seeing that this fraction $\frac{6000+30R}{100}$ is equal to 65, it shall be also equal to $\frac{65}{1}$, and therefore there will be the same proportion of the numerator 6000 + 30 R to the denominator 100, as of the second numerator 65 to 1. Therefore the product under the extreams 6000 + 30 R is equal to the product of the means 6500) take away therefore from both parts 6000, and there will remain an equation between 500 and 30 roots, and therefore divide 500 by 30, the number of the greatest character, you shall have the value of the root $16\frac{2}{3}$ for the pounds of silver mingled by the Goldsmith in the Crown.

Mark well this kind of reduction once for all.

Sect.

Sect. II. *Questions resolved by an* Equation *compounded.*

Question I.

TO *divide* 8 *into two such numbers as their squares being added together, may make* 34.

I put for the first 1 R, therefore the second shall be 8 — 1 R, their squares are 1 Q, and 64 + 1 Q — 16 R, which added together, do give for their sum 64 + 2 Q — 16 R, the question imports that the sum of the squares is 34. Therefore there is an Equation between 64 + 2 Q — 16 R, and 34, which being reduced by addition and substraction, there will remain also an Equation between 2 Q and 16 R — 30, and the whole divided by 2, which is the number of the greatest Cossick Character, there will yet remain an Equation between 1 Q and 8 R — 15, from which I extract the root, as hath been shewn in the fifth Section of the second Chapter. The half of the root is 4, his square is 16, from which take the absolute number 15, rest 1, whose square root 1 added to the half of the number of roots, gives for its sum 5, which is the value of the root; therefore the two numbers sought shall be 5 and 3.

Question II.

To finde two numbers whose product may be 12, and the difference of their squares 32.

I put for the one of them 1 R; therefore seeing that the product is 12, the other number shall be $\frac{12}{1R}$ (for if the product of two numbers be divided by one of those two numbers, the quotient shall be the other number) their squares are 1 Q and $\frac{144}{1q}$, whose difference is $\frac{144}{1q}$ — 1 Q equall to 32, as appears by the question; therefore there will be an equality between $\frac{144}{1q}$ and 32 ✠ 1 Q, and therefore if we make the reduction, as in the third Question of the first Section, we shall also finde an Equation between 144 and 32 Q ✠ 1 QQ, also between 144 — 32 Q and 1 QQ; it behoveth then to extract the square root of the number 144 — 32 Q. The half of 32 is 16, whose square is 256, to which adde 144, makes 400, whose square root is 20, from which take the half of 32, to wit 16, there remains 4. See here the square root, but seeing that the squared square root ought to be taken, I take again the root of 4, and I finde 2 for the value of the root. Therefore seeing that the second number hath been put $\frac{12}{1R}$ the same second number shall be $\frac{12}{2}$, that is to say *6*.

Quest.

Question III.

Two Merchants joyn in company, and together bring in the ſum of 165 Crowns; but the firſt mans money hath been expoſed twelve moneths intire, and the ſecond mans money onely eight moneths: it happens that they gain but 28 Crowns, which added to 165 make 193, which they diſtribute to one another in ſuch ſort, as the firſt takes 67 Crowns, as well for his principal money as for his profit, and the ſecond takes 126 Crowns, the queſtion is what each of thoſe Merchants brought into ſtock.

I put for the money of the firſt man 1 R, therefore ſeeing that the ſum of both was 165 Crowns, the ſecond money is 165 — 1 R. Now if you take away 1 R, which is the ſum the firſt man brought in, from the ſum he received, which was compounded of the principall and profit, you will finde that the firſt mans profit will bee 67 — 1 R, and by the ſame Argument you ſhall finde, that the ſecond mans profit will be 1 R — 39. Now you muſt finde what one root gaineth in eight moneths, which will be done by the Rule of Three, thus, If in 12 moneths there be gained 67 — 1 R, how much will there be gained in 8 moneths, and the fourth number ſhall be $\frac{134}{3} - \frac{2R}{3}$ for the firſt mans profit in 8 moneths; after that I ſeek what the ſecond man hath gained by another operation of the Rule of Three, ſaying; If 1 R gain $\frac{135}{3} - \frac{3R}{3}$ what will 165 — 1 R gain; and I finde for my fourth

fourth number $\dfrac{7370 + \frac{2}{3}\,Q - \frac{464}{3}\,R.}{1\,R}$ which is equal to the ſecond mans profit, which we have already found to be 1 R — 39, and therefore by reduction there will be an equation between 1 Q — 39 R and $7370 + \frac{2}{3}$ Q — $\frac{464}{3}$ R, all which 39 R being added, and $\frac{2}{3}$ Q taken away, there will be an equation between 7370 and $\frac{1}{3}$ Q + $\frac{347}{3}$ R, and conſequently between 7370 — $\frac{347}{3}$ R and $\frac{1}{3}$ Q, and therefore multiplying all by $\frac{3}{1}$, which is the number of the greateſt character, there will be yet an equation between 1 Q and 22110 — 347 R, out of which the ſquare root muſt be extracted. The half of the number of roots is $\frac{347}{2}$, whoſe ſquare is $\frac{120409}{4}$, which added to 22110, make $\frac{208849}{4}$, whoſe ſquare root is $\frac{457}{2}$, from which if you take half the number of roots, there will remain $\frac{110}{2}$, that is to ſay, 55 for the value of one root, and was the money the firſt man put in bank; and for that we have found in the purſuit of theſe operations, that the firſt mans profit was 67 — 1 R, that ſame profit will be 67 — 55, that is to ſay 12, by the ſame reaſon the ſecond mans ſtock ſhall be 110, and his profit 16.

Sect. III. *Queſtions reſolved by* ſurd numbers.

Queſtion I.

To divide any given number (as for example 4) *according to mean and extream reaſon,* that is to ſay, *to divide* 4 *into two numbers, in ſuch manner*

as

as that the whole 4 *may bear the same proportion to its greater part, as the greatest part bears to the least.*

I put for the greatest part 1 R, therefore the least shall be 4 — R. Therefore there is the same proportion of 4 to 1 R, as of 1 R to 4 — 1 R, and therefore the square of the middle part 1 Q is equal to the product of the extreams 16 — 4 R, from which I extract the root according to the rule before prescribed. The half of the number of roots is 3, whose square 4, add to 16, makes 20; out of which the square root ought to be extracted according to the precept; but seeing it is no square number, I must content my self by putting the radical signe before it thus, ℞ Q 20, from which I take the half of the number of roots, and I have for Residue ℞ Q 20 — 2, which is the value of the root by which I shall find with facility, that the other part will be 6 — ℞ Q 20. For the proof of this operation, it behoveth that these two parts added together make 4, and that the lesser 6 — ℞ Q 20, being multiplied by 4, make the product equal to the square of the greater part ℞ Q 20 — 2.

Question II.

To divide 8 *into two parts, between which* 2 *may be mean proportional.*

I put for the first part 1 R, therefore the lesser shall be 8 — 1 R, and seeing that 1 R and 2, and 8 — 1 R ought to be proportional, it behoveth that the square of 2, which is 4, be equal to the product

product of the extreams, which is 8 R — 1 Q, and therefore after the reduction, it will be found, that 1 Q is equal to 8 R — 4, whose square root is ℞ Q 12 + 4. For the one part of 8, and for the other part 4 — ℞ Q 12, both the one and the other root do resolve the question, as you will find, if you take the pains to examine it.

From this practice may be framed an universal Canon, which may serve for the resolution of an infinite number of Algebraical Problemes, which may be conceived after this manner. The sum given, which conteineth the two extreams, ought to be distributed into two equal parts, that the square of the half may be taken, from which the square of the mean proportional given, must be taken, and the square root of the Residue added, and taken from the half of the given sum, will shew the two parts sought. As for example, I take the half of 8, which is 4, whose square is 16, from which I take 4, the square of 2, which is the given mean, and there remains 12, whose square root added to the same half, makes 4 + ℞ Q 12, and taken from the same half, makes 4 — ℞ Q 12.

Question III.

To divide any given number (as for example 4) *into three numbers continually proportional, in such sort as that the squares of the extreams joyned together, may be triple the square of the mean.*

I put 1 R for the number of the middle part, then seeing all three ought to make the same sum of

of 4, we shall have for the sum of the extreams 4 — 1 R. Now seeing that of three numbers continually proportional, the square of the sum of the extreams is equal to the squares of the extreams, and to the double of the square of the middle part. I take the square of this sum 4 — 1 R, which is 16 + 1 Q — 8 R, from which I take 2 Q, which is the double of the square of the middle part, and there will remain 16 — 1 Q — 8 R for the sum of the squares of the extreams: Therefore seeing that the condition of the question requireth a triple proportion, there will be an equation between 16 — 1 Q — 8 R and 3 Q, add therefore 1 Q on both sides of the equation, and you shall have 4 Q equal to 6 — 8 R, and dividing the whole by 4, which is the number of the greatest character, the equation will be 1 Q equal to 4 — 2 R, whose sqnare root is ℞ Q 5 — 1 for the middle number sought, and the sum of the extreams shall be 5 — ℞ Q 5, which being divided into two parts, by the Canon of the precedent question, in such sort as ℞ Q 5 — 1, be the middle proportional, you will find that the extreams are 2 and 3 — ℞ Q 5, therefore the three numbers are 2 and ℞ Q 5 — 1, and 3 — ℞ Q 5, which all together make 4, and are in continual proportion, and the squares of the extreams are triple the square of the mean.

Sect. IV. *Geometrical questions resolved by Algebra.*

Question I.

There is a piece of ground of a greater length than bredth, whose angles are right angles, and in a triple proportion, and their squares taken together, are quintuple their sum. The sides, the diameter, and the capacity or superficies of that piece of ground is demanded.

Before you attempt the resolution of such questions, you must draw their figures.

I put for the least side 1 R, therefore seeing they are in a triple proportion, the other side shall be 3 R, their squares shall be 1 Q and 9 Q, which added together make 10 Q, which ought to be quintuple, the sum of the numbers. Now the sum of the numbers is 4 R, and its quintuple 20 R, and by consequence, see here an equation between 10 Q and 20 R, which are two collateral Characters, and therefore dividing 20 by 10, which is the number of the greatest character, you shall find 2 for the least side, therefore the greatest side shall be 6. Therefore the superficies shall be 12, and the diameter ℞ Q 40.

Question II.

There is an equilateral triangle, whose superficies is ℞ Q 243. *The side and perpendicular is demanded.*

Suppo-

Supposing that the perpendicular of an equilateral triangle doth alwayes divide the side into two equal parts, I put for the half of the side divided, 1 R; therefore the side shall be 2 R. Now seeing that in every equilateral triangle; the square of the side is equal to the square of the perpendicular joyned to the square of the half of the side, which is 1 Q of the square of the whole side, which is 4. I shall have 3 Q for the square of the perpendicular, and so ℞ Q 3 Q shall be perpendicular, which is multiplied by the half of the side, which is 1 R (reducing it first to its square, because of the radical signe which is in the number multiplied) you shall have ℞ Q 3 QQ for the superficies of the triangle: therefore there will be an equation between ℞ Q 243 and ℞ Q 3 Q Q, and therefore there will be also an equation between their squares, which are 243 and 3 QQ, and the whole being divided by 3, the number of the greatest character, there will be yet an equation between 81 and 1 Q Q. I extract therefore the squared square root of 81, and have 3 for the half of the side, 6 for the side, ℞ Q 27 for the perpendicular, and ℞ Q 243 for the superficies of the triangle.

Question III.

There is a Semicircle, whose Diameter is divided according to mean and extream reason, on which there is raised a perpendicular produced to the circumference, and the lesser line which is drawn from the extremity of the diameter, to this point of the

circumference, is ℞ Q 20 — 2, the quantity of the diameter, and of its parts, and of this perpendicular is demanded.

To resolve this question, it is presupposed that the greater part of the diameter shall be equall to the given line, as may with facility be Geometrically demonstrated. That being done, I put for the lesser part of the diameter 1 R, therefore seeing that the other part is given ℞ Q 20 — 2, the whole diameter shall be ℞ Q 20 — 2 + 1 R, which multiplied by 1 R, giveth for the product ℞ Q 20 — 2 R + 1 Q, equal to the square of the given quantity, which is 24 — ℞ Q 320, and by due transposition you shall find 1 Q equal to 24 — ℞ Q 320 + 2 R — ℞ Q 20, from which the square root ought to be extracted, taking exact notice that the particles which have the Cossick Characters may hold place, with the number of roots. I consider therefore in this term of the equation, the number of roots which is 2 — ℞ Q 20, whereof I take the half, which is 1 — ℞ Q 5, to whose square 6 ℞ Q 20, the absolute number 24 — ℞ Q 320, ought to be added, and the sum will be 30 — ℞ Q 500, whose square root ought to be extracted as from Apotomes, as hath been shewn in the last Section of the fourth Chapter, that root is 5 — ℞ Q 5, which added to the half of the number of roots 1 — ℞ Q 5 gives for the sum 6 — ℞ Q 20, which is the value of 1 root, that is to say, of the lesser part of the diameter; and therefore if you add it to the greater part, you shall have 4 for the quantity of the whole diameter, from whence the perpendicular

cular is easily known, provided the rules of Geometry be in any reasonable manner understood.

Sect. 5. *Questions resolved by the* second Roots.

Question I.

THree men have amongst them a sum of money: The first saith to the second, If you deliver me the half of your mony, I shall have 100 *Crowns: The second saith to the third, If you deliver me* $\frac{1}{3}$ *of your money, I shall have* 100 *Crowns: The third saith to the first, If you deliver me* $\frac{1}{4}$ *of your money, I shall have* 100 *Crowns. I demand how must money each one hath.*

I put for the first mans money 1 R of Crowns, and for the second mans money 1 A, and for the third mans money 1 B: therefore the first which hath 1 R with $\frac{1}{2}$ of the second mans money shall have 1 R + $\frac{1}{2}$ A equal to 100, and by consequence $\frac{1}{2}$ A shall be equal to 100 — 1 R, and multiplying the whole by 2, 1 A shall be equal to 200 — 2 R. I begin again therefore the operation, and in stead of 1 A, I put for my second number 200 — 2 R. Now the question requires that the second man (with $\frac{1}{3}$ of the third mans) shall have 100; therefore there will be an equation between 200 — 2 R + $\frac{1}{3}$ B, and between 100, add to both parts of the equation 2 R, and take away 200, there will yet remain an equation between $\frac{1}{3}$ B and 2 R — 100, and multiplying the whole by 3, you will have 1 B equal to 6 R — 300: That being found, I begin again the work,

 and

and in stead of 1 B, I put for the third number 6 R — 300, which added to $\frac{1}{4}$ of the first mans, makes $\frac{25}{4}$ R — 300, which ought to be equal to 100, therefore if you add on both parts 300, there will be an equation between $\frac{25}{4}$ R and 400, therefore if divide 400 by $\frac{25}{4}$, you shall find 64 for one root; therefore the second, which had 200 — 2 R, shall have 200 — 128, that is to say 72, and the third shall have 84. Those three numbers do perfectly satisfie all the conditions of the question.

Question II.

Two men divide between themselves three hundred Crowns in such sort, as that the second mans money divided by that of the first mans, makes $\frac{3}{2}$. It is demanded, how much each of them hath.

I put for the first mans money 1 R, and that of the second 1 A, there is therefore an equation between 1 R + 1 A, and 300, and therefore 1 A is equal to 300 — 1 R, therefore $\frac{300-1R}{1R}$ is equal to $\frac{3}{2}$, and by consequence in cross multiplying these two fractions, I shall have 600 — 2 R equal to 5 R, and dividing 600 shall be also equal to 5 R, and dividing 600 by 5, I shall find 120 for the first mans money, so shall the other have 180.

Question III.

Two find two numbers whose product may be 10, and the sum of their squares 29.

I put for the first 1 R, and for the other 2 A, the product is 1 R A, equal to 10; therefore in dividing the whole by 1 R, there will be an equation between 1 A and $\frac{10}{1R}$, and therefore I begin again the operation, and put for the first 1 R, and for the second $\frac{10}{1R}$, their squares are 1 Q $+ \frac{100}{1q}$ equal to 29, and after the reductions and extractions of the roots, I find 5 and 2 for my sought numbers.

Sect. VI. *Questions resolved* indefinitely.

THat question is said to be resolved indefinitely, in which the numbers are demonstrated in such Algebraical terms, as do satisfie all the conditions of the question proposed.

Question I.

To divide 12 *into four numbers Arithmetically and continual.*

I presuppose that when there is four numbers in Arithmetical proportion, the sum of the extreams is alwayes equal to the sum of the means: whence it follows, that in our question the sum of the extreams shall be *6*, and the sum of the means also *6*. I put for the second 1 R, therefore the third shall be *6* — 1 R, their difference is 2 R — *6*, presupposing 1 R to be the greater number of the two; if therefore I add this difference to 1 R, I shall have 3 R — *6*, and if I take it from *6* — 1 R, I shall have 12 — 3 R, and therefore the four numbers in continued Arithmeticall proportion, shall be

$3R - 6 \mid 1R \mid 6 - 1R \mid 12 - 3R \mid$ and the question is indefinitely resolved, in pursuit of which you may take such a number as you please for the value of 1 R, provided neverthelesss, that you do admit of fained numbers less then nothing: However, if you will have no other numbers than what are real, you ought to take the value of 1 R, beneath 4 and above 2, which will be easily understood by a little experience.

Take for example $\frac{5}{2}$ for the value of 1 R, therefore the first number, which is $3R - 6$, shall be $\frac{15}{2} - 6$, that is to say $\frac{3}{2}$, the second shall be $\frac{5}{2}$, the third $\frac{7}{2}$, and the fourth $\frac{9}{2}$, which added together make 12, and are in continued Arithmetical proportion. And so you may take an infinite number of others.

Question II.

A Vintner hath three sorts of wine, the first is worth 4, the second 6, and the third 10 pence the pint; of these three sorts of wine he desires to fill a vessel which conteins 80 pints, which may be worth 8 pence the pint; I demand how many pints he ought to take of each sort.

You ought here to consider, that the number 80 must be divided into three such numbers, as the first multiplied by 4, the second by 6, and the third by 10, the sums of the three products added together may make 640 (because that all the wine which shall be in the vessel to be filled will cost 640 pence, seeing that if one pint be worth 8 pence, 80 pints will be worth 640 pence) I put therefore for the

the third number 1 R, which multiplyed by 10, which makes 10 R, which being taken from 640, doth leave for the residue 640 — 10 R, which is a number containing the first 4 times, and the second 6 times. On the contrary, seeing that the third hath been put 1 R, therefore 80 — 1 R shall make the sum of the first and the second, which multiplyed by 4, will give 320 — 4 R, which being substracted from 640 — 10 R, will leave 320 — 6 R double to the second number, and therefore the second shall be 160 — 3 R. In like manner the same sum 80 — 1 R multiplied by 6, will produce 480 — 6 R; therefore if you take away 640 — 10 R, there will remain 4 R — 160, double to the first, and therefore the first shall be 2 R — 80. See here the Question resolved indefinitely, the first number is 2 R — 80, the second 160 — 3 R, and the third 1 R. The terms between which you ought to take the value of 1 R are $53\frac{1}{3}$, and 40; if therefore you take 46 for the value of 1 R, you shall have 46 pints of Wine of 10 pence the pinte, 22 of 6, and 12 of that of 4 pence the pinte.

Here I intreat you to consider, That it is impossible perfectly to understand the Rule of Alligation, without the knowledge of *Algebra*: For if you propose this question to one skilful only in Arithmetick, he will give you for the 3 sought numbers, 40, 20, and 20. And if you tell him, that of the sort of Wine of 4 pence the pinte, you have but

A Speciall Note.

16

16 pints, he will remain aſtoniſhed; whereas by your queſtion reſolved indefinitely by *Algebra*, you will be able to give ſatisfaction to this condition infinite kinds and manners.

Queſtion III.

Two numbers are ſought, which have 56 for the difference of their Cubes, and which added together make 6.

I put for the difference of thoſe numbers 1 R, and if from the difference of the two Cubes you take the Cube of the difference of the ſides, dividing this reſidue by the triple of the difference of the ſides, you have for the quotient the product of the ſides, it followes, that if from 56 you take 1 C, and that the reſidual 56 — 1 C be divided by the difference of the ſides, which is 1 R, the quotient $\frac{56-1C}{1R}$ ſhall be triple the product of the ſides; and therefore if you divide this quotient by 3, you will have for the product of the ſides $\frac{56-1C}{3R}$, which is equivalent to $\frac{56}{3R} — \frac{1q}{3}$, therefore if you divide 6, (which is the ſum of your two ſought numbers) into two parts, whoſe product may be $\frac{36}{3R} — \frac{5q}{3}$, you will have the queſtion reſolved indefinitely. Now to attain this, I have given you a Canon in the ſecond queſtion of the third Section, the half of the ſum is 3, whoſe ſquare is 9, from which if you take the product found, there will remain $9 + \frac{1q}{3} — \frac{56}{3R}$, whoſe ſquare root added, and taken from the half of the ſum, gives for the reſolution $3 + R\!\!\!/\ (9 + \frac{1q}{3R} — \frac{56}{3R})$ which

which ſhall be the greateſt ſought number, and $3 - ℞Q\,(9 + \frac{19}{3} - \frac{56}{3R})$ which ſhall be the leaſt. Now theſe two numbers reſolves the Queſtion indefinitely, in ſuch ſort, that if you take two for the value of the root, you will finde that your two ſought numbers ſhall be 4 and 2, and every other number (taken for the value of 1 R) above 2, will reſolve the Queſtion.

Appendix

Appendix.

Questions in *Algebra*, most of which require the Rule of Three in their Operation.

Question. I.

A Merchant receives in exchange for 568 *Crowns, four kinds of Money; of the first* 7 *make one Crown, of the second* 18, *of the third* 21, *and of the fourth* 28 *make one Crown. Moreover, he received of each sort of Money a like number. I demand, How much he received of each kind of Money?*

Put 1 R for the quantity of each sort of Money, and then constitute the Rule of three after this manner.

						Crowns.
If	7 18 21 28	Pieces of money be worth	Crown 1 what is	money 1 R	I answer	$\frac{1}{7}$ R $\frac{1}{18}$ R $\frac{1}{21}$ R $\frac{1}{28}$ R

For if 7 pieces of money make 1 Crown, 1 R of money will make $\frac{1}{7}$ R of Crowns, and so of the rest. Now these Fractions described in the fourth place, do make together $\frac{71}{252}$ R of Crowns, equal to

to the number of 568 Crowns. Divide therefore 568 by $\frac{71}{252}$ and you have for the value of 1 R 2016, and so many pieces of each kind of money he received: which thus I prove.

$$\text{If}\left\{\begin{matrix}7\\18\\21\\28\end{matrix}\right\}\text{Pieces are worth}\left\{1\text{ Crown,}\right\}\text{what}\left\{\begin{matrix}\text{Pieces}\\2016\\\text{Answer}\end{matrix}\right\}\overset{\text{Crowns.}}{\left\{\begin{matrix}288\\112\\96\\72\end{matrix}\right.}$$

For 2016 pieces of the first kind of money do make 288 Crowns, and as many of the second kind make 112, as many of third kind make 96, and of the fourth kind 72, which added together, do make 568 Crowns.

Question. II.

A *Certain man hath two measures of Wine, the one worth* 12 *Crowns, the other* 15. *Now he desires of both these Wines to fill another equal measure, whose worth may be* 13 *Crowns. I demand what part of each of those wines he must take, to fill the other to be worth that price?*

Put 1 R for the part of the measure of the worst wine, and for the part of the measure of the best wine 1 — 1 R, then work by the rule of Three thus.

	Meas.	Cro.		Crowns
Worst wine	1	12	1 R ?	make 12 R
Best wine.	1	15	1 — 1 R ?	make 15 — 15 R

For if one measure of the worst wine be worth 12 Crowns, 1 R of one measure of the same wine will

will be worth 12 R of Crowns, and if one measure of the best wine be worth 15 Crowns 1 = 1 R of a measure of the same wine will be worth 15 — 15 R. Therefore 1 R measure of the worst wine, and 1 — 1 R measure of the best will be worth 15 — 3 R Crowns, which ought to be equal to 13 Crowns. Add therefore 3 R to each part of the equation, and the equation will be between 3 R + 13 and 15; take away therefore 13 from both sides, and the equation will be between 3 R and 2. Divide therefore 2 by 3, and you shall have $\frac{2}{3}$ for the value of 1 R, and so much ought to be taken of the measure of the worst wine, and $\frac{1}{3}$ part of the measure of the best wine, which thus I prove by the rule of Three.

	Meas.	Cro.		Crowns	
Worst wine	1	12	$\frac{2}{3}$?	worth 8	13 Cr.
Best wine	1	15	$\frac{1}{3}$?	5	

For $\frac{2}{3}$ of a measure of the worst wine is worth 8 Crowns, and $\frac{1}{3}$ of the measure of the best wine is worth 5 Crowns, which added together make three.

Question III.

I Have a measure of wine worth ten Crowns; How much water must I mix with one measure, that a mixed (like) measure may be worth seven Crowns?

Put for the measure of water 1 R, then frame the question by the Rule of Three, thus.

Meas.

Meas. Meas.		Meas.	
Wine, water,	Crowns	mixt	Crowns
$1 + 1R$	10	1 ?	worth $\dfrac{10}{1 + 1R}$

For if a measure of wine, together with 1 R of a measure of water, be worth 10 Crowns, one measure of wine and water mingled together will be worth $\dfrac{10}{1 + 1R}$ and so the equation will be between $\dfrac{10}{1 + 1R}$ and 7, which by crosse multiplication is reduced to 10 and $7 + R$, take away 7 from each part of the Equation, and it will be between 3 and 7 R; divide 3 by 7, and you have for 1 R, $\frac{3}{7}$, and so much water ought to be mingled with a measure of Wine, that a measure of the mixture may be worth seven Crowns, and is thus proved:

Measure of Wine and water	Crowns	measure	Crowns
$1\frac{3}{7}$	10	1 ?	worth 7

For if 1 of Wine and $\frac{3}{7}$ of Water be worth 10 Crowns, 1 of that mixture is worth 7 Crowns.

Question IV.

There are in a certain vessel 20 measures of Wine, of which each of them are worth 12 Crowns the measure, Now this vessel is filled up with water, and then one measure of this mixture

is

is worth 10 *Crowns, I demand the content of the vessel.*

Put for the measures $20 + 1 R$, then work by the Rule of Three thus:

Measures of Wine	Measures of water	Cro.	Meas. mixt	Crowns
$20 + 1 R$		240	1?	worth $\frac{240}{20 + 1 R}$

For if one measure of Wine be worth twelve Crowns, a measure of wine $20 + 1 R$ measure of Water together will be worth 240 Crowns: therefore one measure mixt will be worth $\frac{240}{20 + 1 R}$ Crowns, so the Equation is found between $\frac{240}{20 + 1 R}$ and 10, which by cross multiplication is reduced to 240 and $200 + 10 R$; take away 200 from both parts, and there will remain an equation between 40 and $10 R$: Divide 40 by 10, and you have 4 for the price of the root, and so many measures of Water were put into the vessel, and therefore the whole vessel contains 24 measures, thus proved: 24 measures Wine and Water worth 240 Crowns, 1 measure? worth 10 Crowns.

Question V.

T*Wo Letter-Carriers belonging to two Cities distant each from other* 140 *leagues set forth towards*

wards one another, at one and the same time; the one travels eight leagues a day, the other six. I demand on what day they shall meet together.

Put 1 R for the day, then work by the rule of Three thus:

Dayes	Leagues	Dayes	Leagues
1	8	1 R?	8 R.
1	6	1 R?	6 R.

The first therefore in 1 R of daies shall travel 8 R of leagues, and the later 6 R, and both of them together will have meas[ured] 14 R of leagues, that is 140 leagues. There is therefore an equation between 14 R and 140. Divide then 140 by 14, and you shall have ten for the value of 1 R. So the tenth day finished, they met together, which thus I prove.

Dayes	Leagues	Dayes	Leagues
1	8	10?	80
1	6	10?	60

For the first in 10 daies went 80 leagues, and the later went 60 leagues, which both together make 140, the distance of the two cities from one another.

Question VI.

A Certain Merchant bought a quantity of wooll, and another quantity of wax, which cost him together 124 Crowns. Now 100 pound weight of wooll cost him 7 Crowns, and 100 pounds of wax cost him 14 Crowns; but the quantity of wooll that he bought was double to the quantity of wax. I demand

mand how many pounds of each sort the Merchant bought?

Put for the wax 1 R, and for the wooll 2 R of pounds, then work by the rule of Three thus.

Pounds	Crowns	Pounds	Crowns
100 wooll	7	2 R wooll ?	$\frac{14}{100}$ R
100 wax	14	1 R wax ?	$\frac{14}{100}$ R

Therefore there will be an equation between $\frac{28}{100}$ R of Crowns, and 124 Crowns. Divide therefore 124 by $\frac{28}{100}$, and you shall have for the value of 1 R 442 $\frac{6}{7}$, and so many pounds of wax he bought, and of wooll 885 $\frac{5}{7}$, which is thus proved.

100 pounds, wooll	7 Crow.	885 poun. $\frac{5}{7}$	62 Cr.
100 pounds, wax	14 Crow.	442 poun. $\frac{6}{7}$	62 Cr.

And so for wooll and wax together he expended 124 Crowns.

Question VII.

One buyes a number of Ells of Velvet, which he selleth again; he buyes 5 Ells for 7 Crowns, and sells 7 Ells for 11 Crowns, and gained on the bargain 100 Crowns. I demand how many Ells of Velvet he bought and sold in all?

Put for the quantity of Ells 1 R, then work by the rule of Three thus.

Ells

Ells	Crowns	Ells	Crowns
5	7	1 R?	$\frac{7}{5}$R
7	11	1 R?	$\frac{11}{7}$R

You see if $\frac{7}{5}$ R of Crowns which he laid out be substracted from $\frac{11}{7}$ R of Crowns which he received, there will remain $\frac{6}{35}$ R of Crowns for the profit. Therefore the equation will be between $\frac{6}{35}$ R and 100 R, divide 100 by $\frac{6}{35}$, and you have $583\frac{1}{3}$ for the value of 1 R, and so many Ells he bought and sold. Thus proved.

Ells	Crowns	Ells	Crowns
5	7	$583\frac{1}{3}$?	$816\frac{2}{3}$
7	11	$583\frac{1}{3}$?	$916\frac{2}{3}$

By which you may perceive there is 100 crowns gotten.

Question VIII.

A Certain man buyes a number of Ells of Velvet, paying 11 *Crowns for* 7 *Ells: Now he sells the whole again after the rate of* 5 *Ells for* 7 *crowns, and lost* 100 *crowns by the bargain. I demand how many Ells he bought and sold in all.*

Put for the number of Ells 1 R, and then work by the rule of Three thus:

Ells	Crowns	Ells	Crowns
7	11	1 R?	$\frac{11}{7}$R.
5	7	1 R?	$\frac{7}{5}$R.

 Then

Then if $\frac{7}{5}$ R of crowns vvhich he received, be substracted from $\frac{11}{7}$ R of crowns vvhich he expended, the loss will happen to be $\frac{6}{35}$ R of crowns. So the equation will be between $\frac{6}{35}$ R of crowns, and 100 crowns. Divide therefore 100 by $\frac{6}{35}$, and you shall have for the value of 1 R, 583 $\frac{1}{3}$ Ells and so many Ells were bought and sold, vvhich thus I prove:

Ells	Crowns		
7	11	583 $\frac{1}{3}$?	916 $\frac{2}{3}$
5	7	583 $\frac{1}{3}$?	816 $\frac{2}{3}$

Where you see he lost 100 crowns by the bargain.

Question IX.

A *Man buyes* 100 *pounds of Wax for* 17 *Crowns. I demand, how many pounds he must sell for one crown, that so on* 102 *crowns he may gain* 18 *crowns.*

Put 1 R for the number of pounds, then work by the rule of Three thus framed:

Crowns	Pounds	Crowns	Pounds,
17	100	102	600
1	1 R	102 + 18?	120 R

For 102 crowns do give 600 pounds, and if for one crown there be given 1 R of pounds, there will be 120 R of pounds given for 102 + 18, which said 120 are equal to 600 pounds, the quanti-

quantity of the Wax sold. Therefore the equation shall be between 120 R of pounds and 600 pounds. Divide therefore 600 by 120, and so you have for the value of 1 R, 5 pounds, and so many pounds are sold for one crown; so as that in 600 pounds 18 crowns may be gained on 102, thus proved.

Pounds	Crowns	Pounds	Crowns
100	17	600?	102
5	1	600?	120

Where in the first example 600 pound made 102, in this it makes 120, that is 102 + 18.

Question X.

A *Man buyes* 100 *pounds of Wax for* 17 *crowns, in disposing of which he loseth* 18 *crowns, on* 102 *crowns. I demand, how many pounds he sold for one crown?*

Put for the number of pounds 1 R, then work by the rule of Three, thus constituted.

Crowns	Pounds	Crowns	Pounds
17	100	102?	600
1	1 R	102—18?	84 R.

So there will be equation between 84 R of pounds and 600 pounds. Divide therefore 600 by 84, and the value of 1 R will be 7 pounds and $\frac{1}{7}$ and so many pounds he sold for one crown, and lost 18 crowns on 102 by the bargain, which thus I prove:

Pounds	Crowns	Pounds	Crowns
100	17	600?	102
$7\frac{1}{7}$	1	600?	84

Where you ſee he laid out 102 crowns for 600 pounds, and inſtead thereof received but 84 crowns, that is 102—18 crowns.

Queſtion XI.

A Certain man agrees with a ſervant for 12 moneths ſervice, for ten crowns and a coat; but at the end of ſeven moneths, he gives him the coat and two crowns. I demand then at what rate he eſteemed the coat.

Put for the price of the coat 1 R of crowns, and ſay by the rule of Three. If 12 moneths require 1 R + 10 crowns, how much vvill one moneth require? and you ſhall finde that it will require $\frac{1\,R + 10}{12}$ of crowns: Again ſay, If 7 moneths require 1 R + 2 crowns, how much will one moneth require, and you ſhall finde it to be $\frac{1\,R + 2}{7}$ as here under appears.

Moneths	Crowns	Moneth	Crowns
12	1 R + 10	1?	$\frac{1\,R + 10}{12}$
7	1 R + 2	1?	$\frac{1\,R + 2}{7}$

There-

Therefore there will be an equation between $\frac{1\,R + 10}{12}$ and $\frac{1\,R + 2}{7}$ seeing both are the reward of one moneth, which equation by croſs multiplication is reduced to 7 R + 70, and 12 R + 24, take away 24 from both parts, and the equation will be between 7 R + 46 and 12 R. Again, take away 7 from both parts, then it will be 46, equal to 5 R; divide 46 by 5 and the price of a root, and ſo of the coat is $9\frac{1}{5}$ crowns, as appears hereunder. The reward of 12 moneths is $19\frac{1}{5}$, and of 7, $11\frac{1}{5}$.

Moneths	Crowns	Moneths	Crowns
12	$19\frac{1}{5}$	7?	$11\frac{1}{5}$.

Queſtion XII.

A Certain Citizen agrees with a ſloathful ſervant for 30 *dayes, that every day he wrought he would give him* 7 *Groats; but for every day that he idled, and wrought not, he was to allow his Maſter five Groats: when the* 30 *dayes were paſt, it happens, that the ſervant was to receive nothing from his Maſter, nor the Maſter from the ſervant. I demand then, how many dayes he laboured, and how many he idled?*

Put 1 R for the dayes of labour, and 30 — 1 R for the dayes of idleneſs, and then frame the rule of Three thus:

	Day	Groats	Dayes	Groats
Labour	1	7	1 R ?	7 R
Idleneſs	1	5	30—1 R ?	150—5 R

Now ſeeing that his work and play came to one reckoning, there will be an equation between 7 R and 150—5 R, adde 5 R to each part, and the equation will be between 12 R and 150. Divide therefore 150 by 12, and you have for the value of 1 R, 12 $\frac{1}{2}$, and ſo many dayes he laboured, and 17 dayes $\frac{1}{2}$ he idled, which is thus proved.

	Day	Groats	Dayes	Groats
Labour	1	7	12 $\frac{1}{2}$	87 $\frac{1}{2}$
Idleneſs	1	5	17 $\frac{1}{2}$	87 $\frac{1}{2}$

Where you ſee the reward is the ſame with the mulct.

Queſtion XIII.

One ſells 20 pound weight, part Saffron, and part Ginger, for 45 crowns; but he ſold 1 pound of Saffron for 3 crowns, and 1 pound of Ginger for $\frac{1}{2}$ a crown. The queſtion is, how many pounds of each ſort he ſold?

Put for the Saffron 1 R of pounds, and for the Ginger 20 — 1 R of pounds, then by rule of Three work thus:

pound

Pound	Crowns	Pounds	Crowns
Saffron 1	3	1 R?	3 R
Ginger 1	$\frac{1}{2}$	20 — 1 R?	$\frac{20 - 1\,R}{2}$

Therefore the equation is between the sum of 3 R of crowns, and $\frac{20 - 1\,R}{2}$ of crowns added together, and 45 crowns, now that sum is $10 + \frac{5}{2}\,R$ of crowns (for $\frac{20 - 1\,R}{2}$ is equal to $10\frac{1}{2}R$, to which if you add 3 R of crowns, it makes the sum $10 + \frac{5}{2}\,R$) therefore the equation shall be between $10 + \frac{5}{2}\,R$ of crowns, and 45 crowns. Take away 10 from both parts, and it will be between $\frac{5}{2}\,R$ and 35. Divide therefore 25 by $\frac{5}{2}$, and you shall have 14 for the value of 1 R. And so many pounds of Saffron were sold, and 6 pound of Ginger, which thus I prove.

Pound	Crowns	Pounds	Crowns
Saffron 1	3	14?	42
Ginger 1	$\frac{1}{2}$	6?	3

Where you see that the price of 14 pound Saffron, and 6 pound Ginger added make 45 crowns.

Question

Question XIV.

A certain Trades-man hath 2 sorts of coyn, in number 560 pieces, worth 160 crowns; a certain part thereof is worth each piece $\frac{1}{3}$ of a crown, and each piece of the rest $\frac{1}{4}$ of a crown. I demand the number of the first and later sort of money?

Put 1 R for the first, and 560 — 1 R for the later, and then constitute the Rule of Three after this manner.

Money	Crown	Money	
1	$\frac{1}{3}$	1 R?	$\frac{1}{3}$ R
1	$\frac{1}{4}$	560 — 1 R?	$\frac{560 - 1R}{4}$

The fourth number found is equal to 160 crowns, and the sum of their numbers makes 140 $+ \frac{1}{12}$ R (for $\frac{560 - 1R}{4}$ is equal to 140 $+ \frac{1}{4}$ R, to which if you add $\frac{1}{3}$ R, makes the summe 140 $+ \frac{1}{12}$ R.) There is therefore an equation between 140 $+ \frac{1}{12}$ R, and 160 crowns. Take away 140 from both parts, and then the equation is between $\frac{1}{12}$ and 20. Divide therefore 20 by $\frac{1}{12}$, and you have 240 for the value of 1 R, and so much money there was of the first sort, of which each piece was worth $\frac{1}{3}$ of a crown of the later kind 320, each worth $\frac{1}{4}$ of a crown. Thus proved.

Money

Money		Money	Crowns
1	$\frac{1}{8}$	240 ?	80
1	$\frac{1}{4}$	320 ?	80

Where you see the numbers in the fourth place make 160 crowns.

Question XV.

In the Army of the Emperour, the number of the Infantery were octuple to the number of the Cavallery, among them there is distributed 392000 crowns, so as that every Foot Souldier had 5 crowns, and every Horseman 16. The question is, Of how many Horsemen the Army consisted, and of how many Footmen.

Put 1 R for the Horsemen, and 8 R for the Foot, according to the condition of the question, and then constitute the Rule of Three thus:

	Crowns	Horsemen	Crowns	
Horse 1	16	1 R	16 R	56 R
Foot 1	5	Foot 8 R	40 R	

Therefore 56 R of crowns shall be equal to 392000 crowns: wherefore divide 392000 by 56, and you shall have for 1 R 7000 for the number of Horsemen; therefore the Foot shall be 56000, eight times as many, and so there will be distributed to the Horsemen 112000 crowns, and to the Foot 280000, which together make 392000 crowns.

Questio

Queſtion XVI.

A man hath a certain ſum of money in a purſe, which a ſtander by judgeth to be 600 crowns, whoſe errour he thus corrects. If to what I have in this purſe, there be added $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, and from the ſum there be ſubſtracted $\frac{1}{12}$ part of my money, then I ſhould have 600 crowns. The queſtion is, How many crowns he had in the purſe!

Put 1 R for the number of crowns. If the parts $\frac{1}{2}$ R, $\frac{1}{3}$ R, and $\frac{1}{4}$ R, which together make 1 $\frac{1}{12}$ R, be added to 1 R, the whole makes 2 $\frac{1}{12}$ R; take away $\frac{1}{12}$ R, and there will remain 2 R equal to 600. Divide therefore 600 by 2, and you have 300 for the value of 1 R, and ſo much money was in the purſe. For if you add the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, to wit 150, 100, and 75, it will make the ſum of 625, and taking away $\frac{1}{12}$, to wit 25, the reſt will be the number 600, which reſolves the queſtion.

Queſtion XVII.

A certain Traveller goes 9 miles a day, another Traveller after the tenth day paſt, begins his journey from the ſame place, and goes every day 14 miles. I demand in how many dayes the later will overtake the firſt.

Put 1 R for the number of dayes, therefore the firſt over and above 90 miles, which he hath gone in ten dayes, goeth in 1 R of dayes, beſides 9 R of miles, ſeeing that every day he goeth nine miles. But the latter going 14 miles each day, goeth

goeth in 1 R of dayes 14 R of miles, and because the first of necessity in 1 R of dayes goes as many miles together with 90, which he went in ten dayes as the latter went in 1 R of dayes; sith that then they are to meet together, the equation shall be between 9 R + 90 and 14 R. Take away 90 from both parts, and it will be between 90 and 5 R. Divide therefore 90 by 5, and 1 R makes 18. Therefore in 18 dayes they shall come together. For the first in 18 dayes went 162 miles, which added to 90, he made the first 10 dayes, makes 252 miles, which the later went in 18 dayes.

Question XVIII.

A Traveller goes nine miles a day, another Traveller after the end of ten dayes begins the same journey. I demand, how many miles a day the latter ought to travel, that so in 18 *dayes he may overtake the first.*

Put 1 R for the miles. Therefore in 18 dayes he will have travelled 18 R of miles. Seeing therefore that the first travelling every day nine miles, went in 18 dayes 162 miles, and adding thereto 90, which he went the first ten dayes before the second set forth, it is manifest, that he travelled 252 miles. Therefore the equation is between 18 R and 252. Divide 252 by 18, and you have 14 for the value of 1 R, and so many miles the later ought to travell, to overtake the first in 18 dayes.

Question

Question XIX.

A certain man dying, made his Will and Testament, leaving 3000 crowns to be distributed between his Wife, Son, and two Daughters, on this condition, that the portion of the Son might be double to that of the Mother, and the proportion of the Mother double also to the portion of each of the Daughters. The question is, how much each ones portion was?

Put for the portion of one of the Daughters 1 R, for the Mothers portion 2 R, and for the Sons portion 4 R. So there will be an equation between 8 R and 3000 crowns.

The portion of				
one Daughter	1 R	that is	375	Cro.
th'other Daughter	1 R		375	
the Mother	2 R		750	
the Son	4 R		1500	

Divide 3000 by 8, so the value of 1 R will be 375, the portion of one of the Daughters, and therefore the Mothers portion will be 750, and the Sons 1500.

Question XX.

A certain man receiveth of a Merchant, a quantity of Saffron for 10 crowns; and again he receives of the same man 24 pounds more of Saffron, at length he returns to him 30 pounds thereof again, and the Merchant computing the price of the Saffron,

fron, restoreth to him 14 crowns. I demand the price of a pound of that Saffron.

Here you see 10 crowns + 24 pounds to be the whole debt which the Buyer owed to the Merchant, and in like manner 30 pounds — 14 crowns. Therefore there will be an equation between 10 crowns + 24 pounds, and 30 pounds — 14 crowns. Add 14 crowns on both parts, and the equation will be between 24 crowns + 24 pounds and 30 pounds, take away 24 pounds from both parts, and it will be between 24 and 6 pounds. Divide 24 by 6, and you have 4 for the root, and so many crowns one pound of Saffron is worth, which I prove thus. $2\frac{1}{2}$ pounds are bought for 10 crowns, and so the Buyer received of the Merchant $26\frac{1}{2}$, which were worth 106 crowns. If therefore to the Merchant there be restored 30 pounds, the Merchant oweth to the Byer $3\frac{1}{2}$, seeing he received onely $26\frac{1}{2}$ but $3\frac{1}{2}$ pounds are worth 14 crowns, which the Merchant rendered to the Buyer.

Question XXI.

Two men enter into fellowship in Trade, now the second brings with him double the money that the first brings, and 5 crowns over and above. They gain by their Traffick 960 crowns, of which the first takes to himself 300 crowns, and the second 660. I demand how much each put in bank.

Put for the first 1 R, and for the second 2 R + 5, the sum of both together is 3 R + 5, which have gained 960 crowns. Then work by the Rule of Three thus:

3 R

$$3\,R + 5 \quad 960 \quad 1\,R\,? \quad \frac{960\,R}{3\,R + 5}$$

You shall find that the first which brought in 1 R, hath gained $\frac{960\,R}{3\,R + 5}$ which number is equal to 300 crowns which he received. This equation by cross multiplying will be reduced to 960 R, equal to 900 R + 1500. Take away therefore 900 R from both parts, and there will remain 60 R equal to 1500, divide 1500 by 60, and you have 25 for one R, and so much the first put in bank, therefore the second put in 55 crowns, and is thus proved. For both put in 80 crowns.

80. 960 25 ? 300 | 80. 960 55 ? 660

Question XXII.

Three Merchants gain together 700 crowns, which thus they distribute amongst themselves (having regard to the summe each one brought into bank) so as that the portion of the second surmounted the portion of the first by 12 crowns, and the portion of the third surpassed that of the second by 16 crowns. I demand how much each mans portion was.

Put for the portion of the first man 1 R, and then shall the portion of the second be 1 R + 12, and the portion of the third 1 R + 28. These 3 portions together make 3 R + 40, equal to 700. Take away 40 from both parts of the equation, and the equation will be between 3 R and 660. Divide therefore 660 by 3, and you shall have 220 for

for the value of 1 R, and so much was the first mans portion, so the portion of the second shall be 232, and of the third 248, which all together make 700.

Question XXIII.

A Caterer buyes a number of Fowls, so as that if he had bought the $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of the number, and over and above 6, he would then have 100 *just. I demand the number of Fowls he bought.*

Put 1 R for the number, whose $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ make $\frac{47}{60}$ R, which with 6, make $\frac{47}{60}$ R + 6 equal to 100. Take away 6 from both parts of the equation, and the remaining equation will be between $\frac{47}{60}$ R and 94. Divide therefore 94 by $\frac{47}{60}$, and the value of 1 R will be 120, to wit, the number of Fowls that were bought.

For its $\frac{1}{3}$ is 40, and its $\frac{1}{4}$ is 30, and it, $\frac{1}{5}$, 24; which all together with 6, make the sum 100.

Some Examples in *Algebra* concerning *Squares.*

To find two numbers in a given Excess, so as their Squares may have also a given Excess.

LEt there be sought two numbers, whose Excesse is 4, and the Excess of their Squares 144.

Put for the lesser number 1 R, and therefore the

greater number must be 1 R ✠ 4, whose squares are 1 Q and 1 Q ✠ 8 R ✠ 16, their excesse is 8 R ✠ 16, which ought to be equal to 144. Take away 16 from both parts of the equation, and the equation will be between 8 R and 128. Divide therefore 128 by 8, and you shall have for the value of 1 R, 16 for the lesser number, the greate therefore will be 20, that the Excesse may be 4 The squares of those 2 numbers are 256, and 40(whose difference or excess is 144.

Two numbers being given, to find another, with which multiplying both the numbers, makes the greater number a square, and its lesser the side of that square.

LEt the two given numbers be 200 and 5, and let the sought number be put 1 R. Now 1 R multiplied in 200, produceth 200 R, and 1 R multiplied in 5, makes 5 R, which ought to be the side, and so multiplied in it self, ought to make a number equal to 200. But 5 R multiplied in it self, makes 25 Q, the equation therefore is between 200 R and 25 Q. Divide therefore 200 by 25, and the value of 1 R is 8, which multiplied in 200, make the square 1600, whose side is the number 40, which is also produced by the multiplication of 8 in 5.

Some

Some Examples relating to *Cubes.*

To finde a number, which multiplied in it self, and the product multiplied by some given number, may produce a number in a given proportion to the Cube *of the found number.*

LEt the given number be 20, finde another number, which being first multiplied in it self, and then the product multiplyed by the given number 20, may produce a number in a quintuple proportion to the Cube made of the found number.

Put for the sought number 1 R, which multiplied in its self makes 1 Q, which also multiplyed by 20, makes 20 Q, the Cube of 1 R is 1 C, to which 20 Q ought to have a quintuple proportion; so the equation is between 20 Q and 5 C: Divide 20 by 5, and you have for the value of 1 R, 4, the number sought. This 4 multiplyed in it self makes 16, and 16 multiplied by 20, makes 320, which is quintuple to 64, the Cube of 4.

To divide a given number in two parts, so as that their Cubes may make a given sum, which shall be greater than the quarter part of the Cube described of the given number.

LEt the given number be 10, to be divided into two parts, whose Cubes may make 370, which number is greater than the $\frac{1}{4}$ part of the Cube of 10. Put for the first number compounded of 1 R, and half of the given number 1 R + 5, and for the second 5 — 1 R. So these two numbers do make the given number. Their Cubes are C + 15 Q + 75 R + 125, and 125 — 75 R + 15 Q — 1 C; their sum is 30 Q + 250: For + 1 C, and — 1 C, as also + 75 R, and — 75 R, do mutually destroy one another: and of 15 Q, and 15 Q, are made 30 Q: also 125, and 125, make 250. Therefore the equation is between 30 Q + 250, and 370. Take away 250 from both parts, and the equation will be between 30 Q and 120. Divide 120 by 30, and you have 4 for the value of 1 Q, and the value of 1 R, 2. The first part put 1 R + 5 shall be 7; and the second put 5 — 1 R shall be 3. The Cubes of those parts are 343 and 27; which together make 370.

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